Association of Teachers of Mathematics Of Philadelphia And Vicinity

ATMOPAV

An affiliate of NCTM & PCTM

NEWSLETTER
Spring 2015

(PDF website edition)

In this issue:

Keep these dates:

NCTM Regional Meetings:

Atlantic City, NJ • October 21 - 23, 2015

Philadelphia, PA • October 31 - November 2, 2016

PCTM Summer Conference: Lancaster, PA • August 13 - 15, 2015

ATMOPAV Fall Conference

Ursinus College, Collegeville, PA • October 24, 2015

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ATMOPAV

NEWSLETTER

The Association of Teachers of Mathematics of Philadelphia and Vicinity
An Affiliate of the National Council of Teachers of Mathematics and the Pennsylvania Council of Teachers of Mathematics

President's Message

Beth Benzing
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I always feel like I need an extra push to get through the last quarter of the academic year. It is nice when I try something new in the classroom to keep me focused like a project or a new gadget to explore with my students. Putting the project together is time consuming, but it is well worth it in the end, even if it is a hit or a miss. Moving the class outdoors to collect data on the football fied and chalk in the parking lot are old standbys. Utilizing iPads is another way to change up the routine and allow for student-centered, fun approaches that allow students to make discoveries or present solutions and ideas in a classroom.

Even though I have not finished the curriculum, I hope that I have conveyed the big ideas of what the students need to know in order to move on and be able to be successful in their next math class. This is the time of year where I consistently have to remind myself to STOP talking and listen to what they know…or don't. I can't squeeze in the last topic ten minutes before the bell rings because I know that I will be the only one who learned the material.

So I encourage you to take this time to try something new. This suggestion could be considered counterintuitive, given a waning level of energy for both you and your students, but a varied approach to instruction this time of year might be the wrinkle you need to keep you and your students sharp. So spring into spring and have a fabulous end of the year.

It is important we recognize that this is the last ATMOPAV Newsletter for which Lynn Hughes will serve as editor. Lynn has dedicated over a decade of her time and commitment to the newsletter, a level of volunteer service to this paper that is unmatched. If you are looking for a new idea for your spring teaching , consider reading one of countless articles Lynn has contributed. They are always fresh and full of information; her ideas seem to be endless. The ATMOPAV community thanks you, Lynn, for your tireless efforts and for sharing your talents with us for so many years.

Editor's reply: My thanks go to the many contributors to this newsletter over the years, not only those who have consistently come through with something new to share – issue after issue – but also those who dipped a toe into the water and sent a single article. As we search for another editor, I hope you will all consider volunteering. It takes some time and some minimal understanding of formatting and layout, but the newsletter and website are ATMOPAV's major links to its membership and beyond. I'll be happy to assist with the transition in the fall. And you'll get a really nice plaque when you retire! – LH

The ATMOPAV Newsletter is published three times a year. Contributions in the form of articles, reviews, teaching ideas, humor, and opinionated essays are welcome. Material for the Fall 2015 issue should be submitted no later than September 15, 2015. Please be sure to include contact information (e-mail and/or telephone) if you are not a regular columnist. Check our website for information about submitting articles after our new editor has been found. If you are interested in becoming the new editor, please contact Beth Benzing, ATMOPAV's president.

The Association of Teachers of Mathematics of Philadelphia and Vicinity was founded more than 60 years ago. It is an organization of mathematics educators who are dedicated to improving mathematics instruction at every level from kindergarten to college. Through its professional meetings, website, and this publication, ATMOPAV supports, informs, and facilitates communication among teachers, pre-service education students, supervisors, and school administrators from the five-county Philadelphia area. Our awards program recognizes excellence among student teachers, novice teachers, and secondary school students who participate in our annual mathematics competition.

To join ATMOPAV, please complete the membership application in this issue.

Call for Columnists

If you're an educator, you have something to share, and this is a good place to do it. We are looking for more writers of regular columns and of single articles. About what, you ask? Consider these possibilities:

- Activities, especially for grades K - 5
- Mathematics software reviews
- Adapting lessons for students with special needs
- Cross-curricular activities that link math with other subjects
- Diary of a first-year teacher
- Recollections of a retired teacher
- Mathematics-related history
- Useful resource books for teachers
- Math games and puzzles (esp. for elementary and middle school)
- Literature connections and book reviews
- Reviews of math-related apps
- Teaching with particular software: Scratch, SketchUp, spreadsheets, Geometer's Sketchpad, etc.
- Working with challenged learners
- Parent Night/ Math Night ideas

. . . *and whatever else you might think of.*

We are eager for material that applies to any level of mathematics education. If you don't want to write but know a colleague who may, please pass the word.

Math-Related Websites for Teachers and Students

Cut the Knot http://cut-the-knot.org ***The Futures Channel*** http://thefutureschannel.com
Making Mathematics http://www2.edc.org/makingmath/default.asp
Don's Math Website http://www.shout.net/~mathman/ ***CoolMath4Kids*** http://www.coolmath4kids.com
JETS website http://www.jets.org/latestnews/JETS_Challenge.cfm
The Virtual Library of Interactive Mathematics http://www.matti.usu.edu/nlvm/nav/vlibrary.html
The Intermath Dictionary http://jwilson.coe.uga.edu/interMath/MainInterMath/Dictionary/welcome/howto.html
Mathematrix http://www.geocities.com/CapeCanaveral/Hangar/7773/
The MacTutor History of Mathematics http://www-groups.dcs.st-and.ac.uk/~history/
Powers of Ten http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html
Mathworld http://mathworld.wolfram.com/ ***RadicalMath*** http://www.radicalmath.org
S.O.S. Mathematics http://www.sosmath.com/ ***Natural Math*** http://www.naturalmath.com/
The Knot Plot Site http://www.cs.ubc.ca/nest/imager/contributions/scharein/KnotPlot.html
Paper Models of Polyhedra http://www.korthalsaltes.com/index.html
Mrs. Glosser's Math Goodies http://www.mathgoodies.com ***Platonic Realms*** http://www.mathacademy.com
The Math Forum@Drexel http://www.mathforum.org/ ***Explore Learning*** http://www.explorelearning.com
Khan Academy http://www.khanacademy.org/ ***XtraMath*** www.xtramath.org
And, of course . . . ***The National Council of Teachers of Mathematics*** http://www.nctm.org/

Notes from the Conference . . .

The ATMOPAV Spring Conference and Banquet is an important event on our association's calendar. This year's Spring Conference and Banquet (which was held at Germantown Academy in Fort Washington) was no exception. There were five stimulating workshops:

TI Solutions for Algebra I Keystones (grades 7 -10)
Learn it 2Day, Teach it 2Morrow (grades preK – 4)
Addressing Misconceptions About Randomness and Probability (grades 6 -12)
Tech Tips for the "Less Paper" Math Classroom (grades 6 – 12)
Persevering in Problem Solving – Finding More Than 1 Solution (grades 9 – 12)

We thank all of the presenters who gave their time to share their ideas and skills. We can't do a conference without you and your colleagues who find time to give us new ideas and challenge our assumptions. We also thank the team that organized the conference: Bob Lochel, Mary Palladino, Sue Negro, Mark Wassmansdorf, Sonya Wassmansdorf, Beth Benzing, and Allison Rader.

The Spring Conference is the time that we recognize a few of the many people who advance mathematics education, render important service to ATMOPAV, take the plunge into a teaching career, and show us how well they are doing a couple of years down their new professional path. This year's awardees were Susan Negro (Past President), Lynn Hughes (Past Newsletter Editor), Robert Lochel (Outstanding Contribution to ATMOPAV), Marian Avery (Outstanding Contribution to Mathematics Education), Val Cilli (Alan Barson Novice Teacher Award), and Cara Sulyok (Mabel M. Elliot Student Teacher Award). Congratulations to all!

Little Folks, Big Ideas

Rich Murray
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The Miquon School

Math Poems

In our first grade classroom, we like to sing songs and recite poetry, especially if it helps us to learn. We often will use a math poem to help illustrate or simply remember a concept. Math poems can be a fun way to learn about a concept, without taking it too seriously. These math poems can be used as fill-in exercises for those times when you have a few minutes before a transition and are not quite sure how to bridge the time. Just pull one of these out of your pocket and have some fun with them! Below are three poems that we like to use – "The Money Song" for remembering pennies, nickels, dimes, and quarters; "Family of Facts" for learning about how fact families work, and "3-D Shapes" for having a quick way to describe three-dimensional shapes. There are lots of math poems available online. A basic search for "math poems for first grade" yielded a good selection for us. Have fun searching!

The Money Song

Penny, penny,
Easily spent.
Copper brown,
And worth one cent.

Nickel, nickel,
thick and fat.
You're worth five cents,
And I know that!

Dime, dime,
Little and thin.
I remember
You're worth ten.

Quarter, quarter,
Big and bold.
You're worth
Twenty-five, I'm told.
Marisa Curtis of First Grade Glitter and Giggles

Family of Facts

CHORUS:
It's a family of numbers
That adds and subtracts.
So we call it a family,
a family of facts.

Take 2 + 3, 5 is the sum.
Think 3 + 2, and the answer will come.
5 - 3, that's 2 if you please.
5 - 2, you can do that with ease.

CHORUS.

Take __ + __, __ is the sum.
(and so on)
Adapted from the song by Carl M. Sherill

3-D Shapes

3-D shapes are fat, not flat.
A cone is like a party hat.

A sphere is like a bouncy ball.
A prism is like a building tall.

A cylinder is like a can of pop.
A cube is like the dice you drop.

3-D shapes are here and there.
3-D shapes are everywhere!

Author Unknown

MIDPOINT

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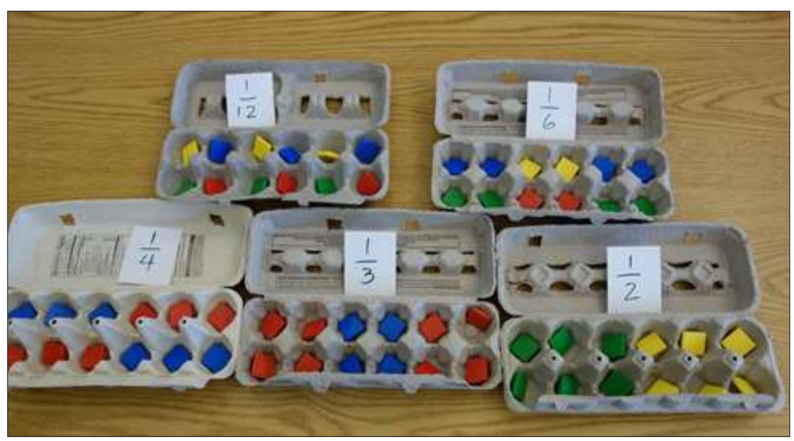
Egg-cellently Useful Cartons Meet Fractions and Base Two

At the ATMOPAV Spring Conference a few weeks ago, I attended a delightful workshop called "Learn It 2Day, Teach it 2Morrow" which was led by Kim Mueller and Lauren Shiffer, teachers from Lumberton, NJ. Although the age range at which it was targets (pre-K through 4th) is younger than I teach, I was hoping to find some useful ideas that I could scale up if necessary, and I was also looking for things to take back to colleagues who do teach that level.

It was an idea-rich session that featured lots of inexpensive gizmos that can make math livelier and more interactive in the classroom. Egg cartons, numerous dollar store items, hula hoops and swim noodles, plastic 2-piece eggs, and much more were spread out on the tables and shown to be good tools for many different mathematical contexts.

It got me thinking about my own math students and what, in particular, I could create from egg cartons that would support our current topics. One fifth-grade group is working on fractions, trying to get foundation concepts to a secure enough level that we could move forward with what I had hoped to do with them earlier in the year. We had been using pattern blocks to move between mixed numbers and improper fractions long with finding common denominators, building fluency with denominators based on 12. (Two yellow hexagon blocks were defined as one whole, so halves, thirds, fourths, sixths, and twelfths are easy to model and exchange.) And, of course, egg cartons typically come with 12 compartments. We have a lot of egg cartons in our classroom because we have a few laying hens in a pen outside, but they are easy to request from students' families. The first thing we did with them in our fraction group was ask students to create models of all of our pattern-block fractions, using square colored tiles to fill the appropriate number of compartments – *e.g.*, pairs of one color for sixths, for example, and trios for fourths:



We didn't use pattern blocks to fill the compartments because we thought it would be confusing to place (for example) three red trapezoids (each of which is worth one fourth) in three compartments and then define those compartments together as showing one fourth. As a result of making the models, students were able to use empty egg cartons to add fractions (using small pebbles that I found sold in bags at a dollar store and bigger bags for fish tanks at a pet shop). Every student was given an egg carton and a supply of pebbles so they could do the examples instead of just watching them be done. Example #1: 1/6 + 1/4 means that we will put two pebbles (1/6) into two compartments and three more pebbles into three more compartments. What fraction of the egg carton is filled with pebbles? What model shape does it match? Yes, it matches only the twelfths model. So we need to record a change our fourths and sixths to twelfths when we write down what we did as an equation: 1/6 + 1/4 = 2/12 + 3/12 = 5/12 . . . and it cannot be simplified.

The next computation did require simplification. Example #2: Let's figure out 1/3 + 1/2. Students referred to the models if necessary and then put four pebbles into compartments and added six more pebbles to six more compartments. They quickly saw and were able to explain that their cartons now held 9 pebbles, and that matched groups they could see in the fourths model. The equation they wrote was: 1/3 + 1/2 = 4/12 + 6/12 = 9/12 . . . and it can be simplified to 3/4.

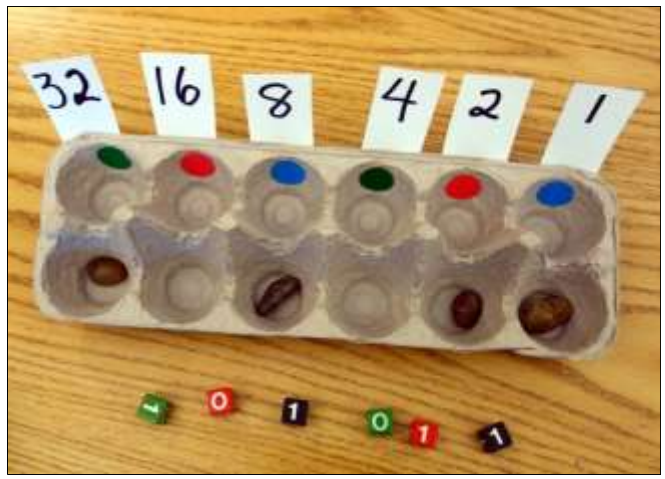
We then gave every pair of students two different fraction dice and directed them to work out the sums using their empty egg carton and the models. They recorded the equations and also used their TI-15 calculators to check their final answers. Because the dice had sides with eighths along with the denominators we had modeled, we agreed to roll again if eighths came up. What happens if the egg carton won't hold the total number of pebbles? Yes, it's an improper fraction that you need to exchange for a mixed number.

The last thing we did in this exploration was invite students to invent a fraction game that could be played with pebbles, cartons, and dice. There were several good ideas that came out of some experimenting, and they played the games happily during the next class period.

This was not an introductory activity, although it could be revised to be one. Several students had already mastered a number-based process for trading unlike fractions and using a common denominator. This activity showed first that they did not fully understand the process in a conceptual sense and then led them to be able not only to do it with the tools but explain it in mathematical terms and link it to their computational process.

The other math group that had some educational fun with egg cartons had begun working in base two. We gave them a puzzle involving containers of different sizes. They drew binary place value charts with the powers of two written out as factor strings ($2^3 = 2 \times 2 \times 2 \times 1$) and reaffirmed the idea that any number to the zero power must equal 1. They extended the chart to the right to show negative exponents and observed the patterns in the fractional places. They were surprised to realize that they hadn't analyzed base ten in these terms recently and drew a lot of valid parallels. They became proficient at the number-guessing game that involves cards with a binary clue in each corner and were able to explain why the numbers on the cards were arranged as they were. (One of the many places on the internet that you can find the cards is here: http://www.mathmaniacs.org/lessons/01-binary/Magic_Trick/.) It was now time for addition and subtraction, and that's where the egg cartons came in.

We got out our binary dice, which came in 3 colors, and they labeled the compartments of their egg carton in alternation, using those same 3 colors (red/blue/green/red/blue/green). We also created value labels for each compartment:



The rules for the 2-player game were simple: No compartment could contain more than one pebble when the player's turn was over. The first person to reach 63 by filling all of the compartments in one row (with the other row being entirely empty) was the winner. Any roll that took the total past 63 was invalid and resulted in a loss of turn. Students rolled a set of 3 dice and put a pebble into each of the correct color-matched compartments in one of the two rows. They rolled again and placed more matching pebbles in the other row. Any place that had a pebble in both columns had to be exchanged for one pebble in the next higher place. After playing this several times, we gave each player a second set of dice with the same 3 colors. Now they could arrange the dice as they wished, as long as the colors of the dice were used to fill a compartment with a matching color. (In the illustration above, a player could have exchanged the 1 in the 32s place with the 0 in the 4s place if it were a better play on that turn.) We also experimented with solving subtraction problems in base two, using the two rows to do regrouping when necessary.

The final activity with the base-two egg cartons was their chance to invent a different game. The only requirements were that it had to involve addition and/or subtraction in base two, use at least one egg carton, and use the dice. Some of the creations were not only educational but hilarious, involving very complicated rules and some physical activity as well.

My thanks go to the two teachers who ran that workshop and got me thinking about egg cartons. Maybe we could use the ones that hold 18 eggs to explore base three . . .

TIMEWASTERS

Don Scheuer
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Please Let It Be Over

The winter, that is. The last couple of months have been as irritating weather-wise as they could be and I, for one, am looking forward to a stretch of about six months of balmy days where the grass is green, the air is a zephyr of delight, and the dimpled spheroids are flying straight and true. I'm not asking too much – am I?

The Winter 2015 issue of *Timewasters* didn't exactly scare too many problem solvers out of the woodwork. Only my dear old friend Hank Field took it upon himself to try one of the puzzlers. He was successful with problem B. He said it was a problem that he had been given when he was 12 years old, and he finally figured it out. His motivation came from substituting another liquid for the water in the problem and actually doing a real time experiment which involved several tastings. Whatever works!

And so, as we move past the snowy equinox into the dearly awaited spring, here are the solutions to the last set of mind benders:

A. **The first term of a sequence is 3. The second term which we shall call "b" is unknown. The sequence is developed by the rule $T_{n+1} = 2T_n + T_{n-1}$, or in other words the next term is found by doubling the preceding term and adding this result to the term before that. If the tenth term of the sequence is 8119, what is b ?**

Answer: The sequence is developed as: 3; b; 2b + 3; 5b + 6; 12b + 15; 29b + 36; 70b + 87; 169b + 210; 408b + 507, etc. The tenth term is then 985b + 1224 and this equals 8119. So, b = 7.

B. **An eight gallon jug is full of water and both a three gallon jug and a five gallon jug are empty. Without using any other containers, divide the water into two equal amounts.**

Answer:	3 – gallon jug	5 – gallon jug	8 – gallon jug
At start	0	0	8
Pour 5 from 8 into 5	0	5	3
Pour 3 from 5 into 3	3	2	3
Pour 3 from 3 into 8	0	2	6
Pour 2 from 5 into 3	2	0	6
Pour 5 from 8 into 5	2	5	1
Pour 1 from 5 into 3	3	4	1
Pour 3 from 3 into 8	0	4	4

(With all due respect to Hank, he took an extra step, but I think it was deliberate.)

C. **(An old classic). A student asked her math teacher, "How many children do you have and how old are they?" The teacher replied, "I have 3 girls. The product of their ages is 72 and the sum of their ages is the same as the number on the classroom door." The student did some calculation and said" There are two solutions." "Yes," said the teacher, "but I still hope that the oldest will someday win a math prize at this high school." The student then knew the ages of the three girls. What are they?**

Answer: Since the oldest is high school age or younger, it is reasonable to consider only those factors of 72 for which the largest is less than 18. The possibilities are:
12 x 6 x 1, 12 x 3 x 2, 9 x 8 x 1, 9 x 4 x 2, 8 x 3 x 3, 6 x 6 x 2 and 6 x 4 x 3. The only two of these choices which give the same room number are 8 x 3 x 3 and 6 x 6 x 2. Since the teacher used the word "oldest", the solution must be an 8-year old and two 3-year olds.

Happy spring! Jump right into these little challenges while the crocuses pop!

A. Thirty-one books are arranged from left to right in order of increasing prices. The price of each book differs by $2 from that of each adjacent book. For the same price as the book at the extreme right, a customer can buy the middle book and an adjacent one. Where in relation to the middle book is the adjacent one?

B. Three fair dice are tossed. What is the probability that the three numbers showing can be arranged to form an arithmetic progression with common difference 1?

C. If n is a three-digit number whose units' digit is a 3, find the probability that n is divisible by 3. (No, the hundreds' digit cannot be 0!)

Send your solutions to Don at his email address above.

A Snow Day Idea

Yes, Don, winter has ended, but we all know it's going to come back again next year. So save this clever activity from our board president, Beth Benzing, for the next time your school is closed due to white stuff.

STATISTICS SNOW DAY ASSIGNMENT

1 point bonus
Create a normal curve out of snow and take a picture. Please send to me via email.

25 points
Collect a data set with at least 30 values. The data must involve snow. Examples: Measure the height of 30 snowmen, throw a snowball and see how far you throw it, measure the height of the snow at 30 different spots around your house. During the next week, you will have to describe the distribution by considering the center, shape and spread. Create a distribution of the graph. Feel free to use Fathom or Excel to generate these graphs. Verify that the distribution is normal by using the 68-95-99.7 rule and by creating a normal probability plot. Write a few sentences describing your results obtained by both of these methods. Happy Snow Day!

I assigned this project for an anticipated snow day for my AP statistics class. Students had many wonderful original ideas as they considered the height of the snow at various places around their home, the distance between foots steps made in the snow, the time it took to shovel a certain area in their driveway or the circumference of 30 snowballs. Pictures associated with the one-point bonus problem were not as original.

The second assignment is a modified version of a project originally written by ATMOPAV member, Leigh Nataro.

Book Review:

I See What You Mean: Visual Literacy K – 8 (2nd edition)
Author: Steve Moline

The author makes a strong case for bringing more visual strategies and tools into our programs in every subject area. Although the title suggests that the contents are suitable for middle school and below, there are things that would benefit high school students as well. It is not specifically a book for mathematics teachers; the cover illustrations relate only to science. But inside he makes it clear that visual literacy is required in every subject area and, indeed, throughout our lives. We live in an increasingly information-dense world, and much of it comes to us and can be best interpreted in visual form. When students are asked to represent something visually instead of just with words (*e.g.,* through a Venn diagram), relationships and patterns are often easier to see and may, in fact, be discovered only by taking a visual approach. He notes that many of his examples would be better printed in color. Because that would be prohibitively costly, he makes those illustrations available in color at the publisher's website (www.stenhouse.com/iswym) and identifies them accordingly in his text.

The chapters that follow the introductory material are: Simple Diagrams, Maps, Analytic Diagrams, Process Diagrams, Structure Diagrams, Graphs, and Graphic Design. The book concludes with an extensive list of resources for each chapter: books, websites, and apps. This book would be especially useful to teachers in general classrooms (as opposed to mathematics specialists), as they would have many opportunities to bring these ideas into their students' instruction throughout the day. – *LH*

ISBN-13: 978-1571108401 (ISBN-10: 1571108408)
Published 2011 by Stenhouse – 272 pages

A continuing dialogue on . . .

Pedagogy in the Mathematics Classroom

Editor's note; Although some people would argue that "pedagogy" applies only to teacher-directed, didactic instruction, we choose to interpret it as a label for what we should be doing to make our classrooms joyous, student-focused, centers of durable and independent learning – however that may be achieved. So we invite you to share your methodology, perhaps talk a bit about the population that you serve, and give us some insight into why it works for them.

We're Too Cool to Function!

S. Leigh Nataro
Moravian Academy Upper School

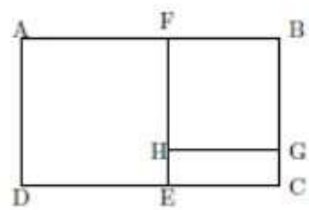
You know that your students are enjoying doing challenging math problems when:
A) They show up 45 minutes before the school day starts.
B) They cheer and give each other "High Fives" when getting a question right.
C) They draw all over your board after the contest is over to discuss how to solve a problem.
D) They design a math t-shirt that says "We're too cool to *f(x)*"
E) All of the above.

The correct answer is E. All of these things happened when my students participated in MAA's Math Madness competition. There are several things that make this competition great.

First, students compete as a team against another school. The team score is made of the top scores, like in a cross-country meet. The top scoring students varied from week to week and sometimes even included freshmen.

Second, students compete against themselves. They see their previous personal best on their computer screen and try to get a new personal best. The problems are varied - some of the problems are multiple choice and some are short answer. Some involve traditional topics in algebra or geometry. Others involve graph theory or probability. But no matter what, there is always something every student can answer and they get immediate feedback about their answer being right or wrong. Here is a sample problem. Can you figure out the correct answer?

Rectangles $ABCD$ and $HGCE$ are similar, $AD = 1$, and $AFED$ and $BGHF$ are squares. What is CG?



(A) $\frac{2-\sqrt{3}}{2}$ **(B)** $\frac{\sqrt{3}-1}{3}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{3-\sqrt{5}}{2}$ **(E)** $\frac{\sqrt{5}-1}{2}$

Third, after the bracket round of competition was over, we could still arrange for competitions against other schools, and my students wanted to continue to compete! The 7:20 AM meeting time was not a deterrent!

Finally, the cost was reasonable at $12.50 per student for the entire fall season.

What is the next step for my team? I have found a set of videos at MAA's Curriculum Inspirations website to help my students develop some new strategies to attack challenging problems. To find them, search the internet for "Curriculum Inspirations by James Tanton."

We also watched a portion of the Who Wants to Be a Mathematician Competition from January 12, 2015 at http://www.ams.org/programs/students/wwtbam/wwtbam-video, and we will be participating in an abbreviated spring round of Math Madness.

To find out more about how to have your students become a part of Math Madness for the fall of 2015, contact Tim Kelley at tkelley@maa.org.

Editor's note: In one of the early "Star Trek" television episodes, Spock says to another Vulcan (with whom he has lost a competition for a mate), "Someday you may discover that having a thing is not the same as wanting it" or words to that effect. I have been imploring Josh Taton for an article about the Philadelphia Area Teachers Math Circle (PATMC) through the course of several issues. How long should it be? he asked. Length is not a problem, I replied in blissful ignorance. So he has finally sent me a novella. As a result, I could print only part of it for mailing, but it is here in its entirety. Thank you, Josh. You are a light in one of the darker corners of our educational universe. Or something.

Philadelphia Area Math Teachers' Circle

Introduction: Believe It or Not, I Do Math for Fun!

"Whaddaya do, go home on the weekends and just do math?"

The Medical Curiosity of Math Teachers

Math teachers get this question a lot. My preteen students asked it most often, but I'll bet their parents and my non-math colleagues wondered silently, too. Reading between the lines, I would hear: "He must not have a life! How could anyone teach math and *enjoy* it?" Outwardly, I would laugh off this question and its companion allegations. Inwardly, I would bristle at the sentiment and that the words to respond were so elusive. *But wait! I've been skydiving, I go to shows, I work out…* My sub-vocal list would go on and on.

As a founder of the Philadelphia Area Math Teachers' Circle (PAMTC), I now have better language to use. "Nah," I would begin, brightly. "I do lots of things when I go home. But, yeah, I also *discover math!*" Such an assertion seems outlandish, but—believe it or not—I do, in fact, *do math* for fun. Most people probably imagine that I sit amidst a pile of dusty textbooks and scribble solution after solution to the "homework problems." This could not be further from the truth, however, as I hope to illustrate.

A Dish Served Cold: Challenges of the American Educational Landscape

To put it bluntly, we Americans are bad at math. Decades of research and volumes of anecdotal evidence have proven it. Very few of us pursue advanced study of mathematics beyond the required courses in high school or college (Snyder & Dillow, 2013, Tables 179 & 331). Consider, also, this dismaying 1980's anecdote from Green (2014): the A&W restaurant chain introduced a new hamburger with one-third pound of beef, but it suffered lackluster sales in comparison to McDonald's smaller-sized Quarter Pounder. Even though the A&W burger was the preferred choice on taste tests, and even though both burgers were priced comparably, focus-group research revealed that most people falsely presumed the Quarter Pounder was a better deal. They reasoned, incorrectly, that the "4" implied within the name "Quarter Pounder" meant the McDonald's burger was larger.

Clearly, standard models of teaching haven't worked, and meaningful reform has proven perniciously difficult for generations (Lortie, 2002; Tyack & Cuban, 1995). Improving teachers' PD is seen as a key ingredient for change, but improvements have been difficult to attain (see Yoon et al., 2007; Ball & Cohen, 1999; Cohen & Hill, 2000; Darling-Hammond & McLaughlin, 1995; Desimone et al., 2002; Heck et al., 2008). We therefore need a *different* form of professional development for teachers of mathematics, and teachers, themselves, agree. Most say, in fact, that "much of the professional development available to them is not useful" (Darling-Hammond et al., 2009, p. 5), especially because of the lack of content-oriented PD. Content-oriented PD concentrates on aspects of teaching and learning unique to a particular area, like history or mathematics. The majority of teachers receive fewer than 16 hours (or two days) of content-oriented PD annually, and this amount is decreasing (Darling-Hammond et al., 2009). Research has shown, in contrast, that 50 hours or more is needed (Yoon et al., 2007, p. 14). One-time workshops, the most common forms of PD in American schools, have virtually no effect on teachers or their students (Darling-Hammond et al., 2009; Yoon et al., 2007).

In addition, changes in the educational landscape—including the Common Core State Standards Initiative—aim to support students' development of critical-thinking and problem-solving abilities and place additional demands on teachers. Teachers also say they are unprepared to implement these sorts of changes (EPE Research Center, 2013). One persistent barrier they encounter is time: U.S. teachers spend about 80% of their workdays on teaching, while their counterparts in other, high-performing nations spend about 60% and have the remainder available for collaboration and professional learning (OECD, 2014, Table D4.1).

The Brownie Problem

"What are you giggling at?" my partner, Andrea, asked. We were driving home to Philadelphia from a friend's birthday party on Long Island.

"I just remembered the *Seinfeld* episode I saw, today. Elaine steals a piece of cake from Peterman's fridge. Turns out the cake was a collector's item: it was from the 1937 wedding of King Edward VIII and Wallis Simpson and was worth $29,000. She tries to hide the missing piece by smoothing out the frosting." I laughed, reminded of the cake we inhaled hours earlier.

"Ha! That's interesting," Andrea said, clearly amused. But I also detected a serious tone.

"Why is that?"

"Well, it reminds me of the Brownie Problem from the Math Teachers' Circle you told me about. I've been thinking about it."

"Yeah, me too," I replied. "Let's see if I can remember the problem…."

At the end of the school term, you bake a tray of brownies to share among your two 7th grade classes. While the brownies are cooling on the counter, some mischievous scamp sneaks into the kitchen, cuts out a rectangular piece, and steals it. The tray of brownies must be split evenly between the two classes, and you have only moments to do so. Is it possible to divide the brownie tray evenly, using a single cut from your knife? Why or why not?

To stay awake on our drive, we chatted about the problem. As I had told Andrea, I wanted to use the problem in an upcoming Philadelphia Area Math Teachers' Circle (PAMTC) workshop, but I wasn't quite sure how to interpret it and I didn't yet grasp possible solutions. So we worked it out together, riffing on each other's questions and ideas.

Different Ingredients in "Doing Mathematics"

The Traditional View: Recipes and Rules

Most American students experience mathematics through models known as direct instruction (Rosenshine & Stevens, 1986) or gradual release (Pearson & Gallagher, 1983). With direct instruction, the teacher sets lesson objectives and then walks students toward these aims; near the end of the lesson, students typically work alone in solving practice problems. In gradual release, the teacher also leads: first, by modeling the steps; then, as the class practices together, by debugging any errors. Finally, as with direct instruction, students complete practice problems on their own. The gradual release model is also known as "I, we, you" because the teaching sequence proceeds from the teacher ("I"), to the class ("we"), to the individual student ("you"). In this article, I portray alternatives to these views, arguing that they are inherently problematic. First, though, I note that these traditional views are so entrenched that, when presented with alternatives, people sometimes dismiss them outright. Teachers employing alternative models are not actually "teaching math"; students of such teachers, the argument goes, are not "learning math"—at least not the math that they need to become "college- or career-ready" (see Rubinkam, 2014).

Problems with the Traditional View

Boaler (2013) argues that this traditional view of math is limiting, especially in that it causes anxiety for students. Being a struggling reader in our society is embarrassing, but it's perfectly acceptable to be math-phobic. She implies that contributing to this phobia are the focus on individual competition and correct answers (rather than collective making sense of ideas) and the focus on skill-and-drill worksheets that students complete in an unthinking way and quickly forget, once completed (rather than meaningful and engaging problems). Boaler also explains that decades of research show it does not have to be this way: "when mathematics is opened up and broader math is taught" (2013, para. 3), she argues, students not only become engaged and interested in math, they also do better at it.

Before painting an alternative picture of what it means to *do math* and what Boaler (2013) refers to as "broader math" (para. 3), I also point out that research has demonstrated schools are contrived and inauthentic arenas for doing mathematics. Studies have shown that students and adults are eminently capable of performing complex calculations, when playing sports or going to marketplaces, but cannot do so when presented with the *exact same* problems on paper tests in classrooms (Herndon, 1971; Lave, 1988; Nunes, Schliemann, & Carraher, 1993). We therefore need mathematical experiences in schools that are meaningful for and understandable to students. I argue that, to achieve this, we need PD experiences that provide the same for teachers.

The Brownie Problem: Part 2

"What are some questions that you have about the Brownie Problem?" I asked Andrea. As she replied, I ticked off her questions from my own mental checklist.

"How big was the piece that was stolen? What shape was it? Where was it cut?" she asked.

"Right! Right!" I replied, excitedly. "I wondered, too—I have no idea! I also wonder how big the tray of brownies is, including how thick it is. And does thickness even matter?"

As we started to ask these questions, we also began to answer them. In other words, we developed a set of assumptions that helped us to interpret the problem and establish boundaries on reasonable approaches.

"Let's start by assuming that the stolen piece is also rectangular and that it's been cut from anywhere in the tray."

"OK, fine. So…."

An Alternative Perspective: Creating New Dishes

The main problem with the traditional view of mathematics and mathematics learning and teaching is that it bears little resemblance to how mathematicians *actually* work. In school, most of us did math the same way that we did cursive or did spelling; that is, we copied and copied the forms and rules established by others. I argue, though, that we received fallacious messages from these practices and that we don't actually know what it means to *do math*.

In contrast, to mathematicians, *doing math* is a creative endeavor. It does not involve following pre-established recipes. Mathematicians are like artists, experimental scientists, critics, politicians, and chefs all rolled into one. They estimate. They mix. They create. They test. When something fails, they scrap their work and start all over again. Often, they work together. And when they do, they create new language, disagree and debate, and build on one another's ideas. In short, they are curious and they pursue lines of inquiry wherever they lead; when doing math, they support and revise their ideas, while also drawing upon the collective wisdom of their predecessors (Lakatos, 1976; Pickering, 1995).

Studying mathematics bears resemblance to studying art or cuisine in one additional, crucial aspect: just as every painting or dish reveals something new each time we view or taste it, mathematical thinkers can discover something new each time they return to seemingly familiar ideas. Ma (1999) noted that teachers in elementary schools in China have a deeper understanding of fundamental mathematical ideas than do their American counterparts. I speculate this is partly because American teachers may be embarrassed to investigate or are simply unaware of the importance of investigating basic ideas of whole number operations, fractions, or decimals. I believe that such reflection would be productive, revealing subtleties either long ago forgotten or otherwise missed.

At its heart, mathematics is both a problem-solving venture and a language. In order to make sense of mathematics, students need to be exposed to problems that are important in their everyday lives, and they need to have opportunities to communicate mathematically—to explain their thinking, to critique others' thinking, and to develop notation and terminology that makes sense to them in the context of solving a problem. This is what Boaler (2013) means by "broader math" and what typifies the work of professional mathematicians (and other critical thinkers!). The Common Core State Standards recognize the importance of active mathematical inquiry, as well, in the eight Standards for Mathematical Practice (NGA Center / CCSSO, 2010). The Practice Standards are easily overlooked but actually represent the foundations of the Common Core reforms and decades of research in math education.

Teachers' Professional Learning: The Old and the New Menus

The Old Menu: Traditional Dishes and Their Problems

As noted above, professional development for American teachers mainly consists of one-time workshops. In most cases, these workshops fail to address learning and teaching within a specific content area, and—with respect to the type, nature, and amount of PD—they rarely align with established best practices (Darling-Hammond et al., 2009). This sort of delivery method, generally speaking, puts teachers in the role of receivers (Darling-Hammond et al., 2009, p. 6). Consequently, the traditional approach to PD replicates problems with traditional approaches to teaching and learning: teachers inevitably become passive, disengaged, and struggle to apply what is being taught to their classrooms. Yoon and her colleagues (2007) explain that the difficulty of translation rests on the highly contextual nature of teaching and learning. In other words, generic PD does not address teachers' questions about *their* classrooms or their students' specific needs.

The New Menu: Alternative, Research-Based Approaches

For these reasons and many more, researchers have argued for a different set of principles to guide PD (Garet et al., 2001; see also Doerr, Goldsmith, & Lewis, 2010). Many of these are general recommendations for improving teachers' learning; in other words, they could apply to teachers in any academic discipline. For example, professional development experiences regarded as high quality are those that provide regular and ongoing opportunities for active learning, for building productive habits of mind, and for collaboration. In this section, though, I argue that PD for teachers of mathematics must also include something *more* and *different*. Math teachers, I believe, face an additional set of challenges that necessitate a novel approach.

First, teachers need to know *much* more mathematics than what they teach. The Conference Board of Mathematical Sciences (CBMS), a blue-ribbon panel of mathematics and mathematics education experts, notes that, across all grades, teachers' mathematical knowledge is often incomplete and superficial (CBMS, 2012). The CBMS recommends, then, that all teachers *continue* deepening and expanding their mathematical understanding while they teach. The panel's recommendations align with the artful vision of mathematics described previously: returning to seemingly basic ideas can reveal complexities previously unnoticed. Research supports regular engagement in mathematical thinking for students (Murayama et al., 2013). Why shouldn't the same principle apply to teachers?

I argue, though, that increasing the quantity of content-oriented PD is not enough, because there simply isn't enough time within the schedule. Requiring teachers to undertake coursework at colleges or universities is not a solution, either: when teachers pursue advanced study, significant improvements in teaching have not occurred (Hanushek & Rivkin, 2006). Graduate coursework, under the pressure of exams and grades, tends to reinforce the outdated models of learning and teaching. Teachers need to understand mathematics, but they also need to understand the models, examples, and representations of mathematics that are helpful for building meaningful and deep connections between ideas (Ball, Thames, & Phelps, 2008). Helping teachers bolster their mathematical knowledge for teaching is, in fact, tied to students' improved achievement (Hill, Rowan, & Ball, 2005; Hill et al., 2007; Jacobs et al., 2007). To accommodate teachers' schedules and to promote higher levels of engagement, I therefore argue for *recreational* professional development in mathematics (much like the playful way Andrea and I worked on the Brownie Problem).

Second, teachers of mathematics require *different* PD opportunities than their peers. This, I argue, is because of the deeply-rooted negative perceptions of mathematics that reside in our culture. Tobias (1978/1995) originally defined math anxiety as the feelings of apprehension that impede our performance on mathematical tasks. Math anxiety also manifests itself when we *avoid* mathematics, perhaps even thinking that it isn't worth our while to explore mathematical ideas. I think that the great majority of us are anxious about math, including many math teachers, themselves (Hembree, 1990).

In addition, most teachers (as students) learned about math from their own in-school experiences; more likely than not, these experiences were also stressful and uninspiring (Beilock et al., 2010). Teachers also learned about what it means to *do math* from these experiences, which (for many) means copying down a set of rules into notebooks and performing a bunch of practice problems to follow the rules—even if the rules don't make sense. Consider, for example, a familiar saying about dividing fractions: "Don't ask why, just flip and multiply!" Why would we expect this chain to break spontaneously? Todays' teachers also face intense pressure to increase students' test scores, to proceed rapidly through the curriculum, and to meet the new standards. These pressures further decrease the chances that K-12 mathematics will be engaging or joyful.

Not only do we as a society need cultural retraining to break down unhelpful assumptions about mathematics, but our teachers need retraining, too. I was a teacher and I have a great respect for teachers, and so I locate the problem not with teachers but within our culture. Fortunately, research has shown that Boaler's (2013) "broader mathematics" is not incompatible with standardized testing (see Grouws et al., 2013; Tarr et al., 2008). To the contrary, many new state assessments even ask students to demonstrate deep conceptual understanding and an ability to explain their thinking, capacities that would be bolstered by an enthusiasm for mathematics.

The Brownie Problem: Part 3

"So, let's assume that the stolen brownie-piece is a rectangle and let's make it really simple," Andrea said.

"What do you mean by 'simple?'" I asked, wondering where she was going.

"Well, imagine that the thief was really greedy and stole half the tray, cutting it right down the middle." I imagined the following picture (see Figure 1a). *Editor's note: All figures appear at the end of this article.*

"Well, then, it's easy to figure out how to cut the remaining piece in half," I said. "Just cut the brownie in the opposite direction, say lengthwise." (See Figure 1b.)

"Right," she said. Mile after mile of the New Jersey Turnpike drifted away, as we problem-solved.

"So, then," I began, "let's move to something a bit more complicated. Instead of taking half, the thief steals one-fourth of the tray—cutting out an entire corner. In that case, you could just make a diagonal cut and you'd create two equal pieces that were the exact same shape, like a trapezoid." (See Figure 1c.)

"But that's not right," she replied. "Imagine a very narrow, skinny rectangle. With a quarter of the tray removed, just like before. If you cut from one interior corner to the opposite corner of the tray, then your two pieces would *not* be the same size. Or shape!" She demonstrated by drawing in the air (Figure 1d).

"Yeah! So cutting along a diagonal, from one corner to another, won't work. Hmm…."

Teachers as Patrons and as Chefs

Teachers' work, we know, consists of oodles of essential tasks: grading homework, planning field trips with colleagues, contacting students' families, and so on. But when do teachers have the opportunity to grapple with the content they teach? To *practice* the content? To grow, as learners (or patrons) and as *producers* of knowledge (or chefs)?

At first, it may seem odd to regard teachers as producers of knowledge instead of transmitters of knowledge. Most teachers, however, are also practitioners of the content they teach. For example, English teachers often read, deeply, for pleasure. Many even write stories, essays, poetry. Art teachers are often artists, themselves, and many music teachers perform in musical groups. These are all avocations, pursued for fun and often within others' company. Scholars have shown that these pursuits are far from frivolous (e.g., Graham & Zwirn, 2010). Atwell (1987) argues:

> When teachers demonstrate interest and excitement in our fields, we invite students to believe that learning is valuable. We answer the question, "Why do we have to do this?" with our own conviction and passion, modeling the power we derive from our knowledge and experience. (p. 48)

In literacy education, scholars have invited teachers to think of themselves as writers—to *become* writers—not only for the purposes of modeling desired behaviors and dispositions, but also because the act of writing is an authentic way to learn about writing (see Graves, 1983; Calkins, 1986; Atwell, 1987/1998). And teachers cannot merely teach; they must be learners, too. In Hall-McEntee's (1998) words, "Teachers retain their vulnerability as learners, sensitive to the complex challenge of putting thoughts into words [by writing]. Writing is a humble act during which the writer is always up against personal inadequacy. Sharing this struggle with students models genuine learning." The National Writing Project and Cochran-Smith and Lytle (2009) also show us the importance of teachers writing about their teaching practice. In other words, for students to become better readers (and writers), they need to write; for teachers to become better teachers of reading and writing, they need to write, too. And since teaching is, inherently, a communicative endeavor, teachers are well served by writing *about* their teaching.

Therefore, while some scholars in literacy education have recognized the interconnectedness of reading texts and writing (or producing) texts and have promoted the use of new pedagogical models, like writing workshops (Atwell, 1987/1998; Calkins, 1986; Graves, 1983), we have no analogue in mathematics education. Modes of teaching in K-12 mathematics, and students' achievement, remain largely stagnant in the U.S. Yet, we need students to become mathematicians—to become creative and curious mathematical thinkers. To get there, I argue, we need teachers to become mathematicians, too.

I hope I am making clear that neither grading homework, nor completing worksheets, is *doing math*. Really doing math is something else, entirely. Indeed, I helped to start the Philadelphia Area Math Teachers' Circle for one simple reason: to provide a *recreational* space for teachers to engage in, and hence deepen their appreciation of, *authentic* mathematical thinking. I detail how we work in the PAMTC, below, noting that the Brownie Problem has thus far illustrated our model.

The Brownie Problem: Part 4

"Oh! I think I have an idea!" exclaimed Andrea. "What if the missing piece was located entirely within one quadrant of the tray? Then, you could…" She elaborated on her approach.

"That would work!" I replied, smiling. "I wonder, though, what would we do—if the missing piece was located elsewhere in the tray…." After some thought, both of us realized another possible solution.

"Hmm," I began, still unsatisfied. "I wonder if the shape of the brownie tray or the shape of the missing piece matters. Can we find an approach that will work for all types of brownie trays and all types of missing pieces? In other words, for what shapes of brownie trays and missing pieces will our solutions work?" (See Figure 2.)

Days later, long after our trip had passed, we eventually arrived at (and wrote an explanation of) our approach to this extended problem.

Figures and Figure Legends

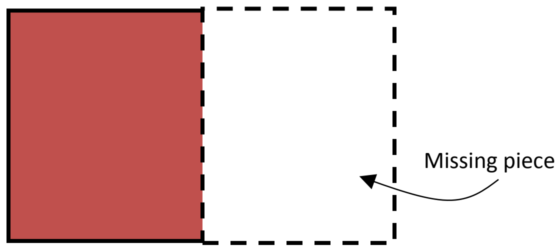


Figure 1a. A brownie tray with half missing

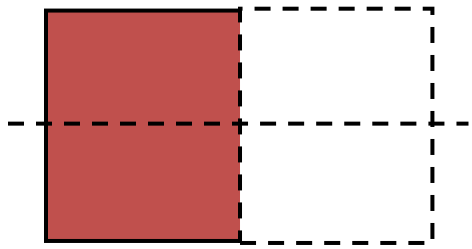


Figure 1b. Cutting the remaining piece in half

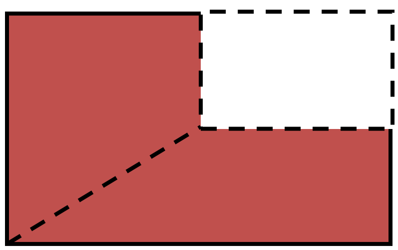


Figure 1c. A fourth of the brownies missing, and the remaining piece cut in half, perhaps?

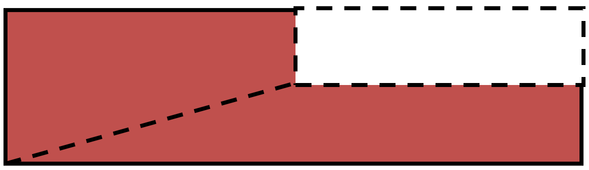


Figure 1d. A long, rectangular tray, and the remaining piece clearly not cut in half

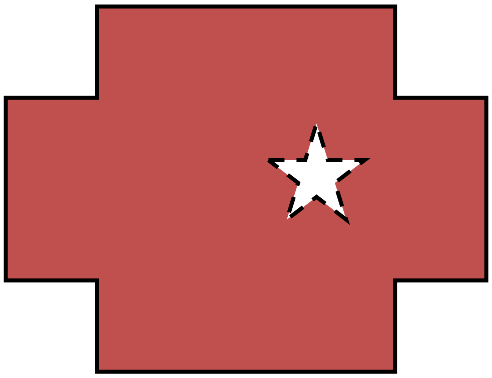


Figure 2. An oddly-shaped brownie tray with an oddly-shaped missing piece

A Fun, New Flavor of Professional Learning: Math Teachers Circles (MTCs)

Math Teachers' Circles: The Model and History

The model, which we follow in our workshops in the Philadelphia Area Math Teachers' Circle and which I describe below, was developed by the American Institutes of Mathematics (AIM) in 2006. The AIM is one of eight mathematical sciences research institutes funded by the National Science Foundation that bring mathematicians together to collaborate on solving cutting-edge research problems. The AIM noted that most mathematical discoveries are made by teams of scholars, working together, debating and discussing. They thought, "Why not find a way to not only show K-12 mathematics teachers how professional mathematicians work, but to also provide them with an opportunity to do real mathematical work, themselves?" And, so, Math Teachers' Circles (MTCs) were born (AIM, 2014). Currently, there are about 60 MTCs across the U.S. with a cumulative total of approximately 1,000 regular attendees (Silverstein, 2014).

At MTC workshops, after an initial greeting, a professional mathematician describes a problem that we then tackle by working together. The professional mathematician remains "hands-off," throughout, letting us grapple with possible solutions—of which there are usually many. This approach contrasts with what happens in most American classrooms, wherein teachers often step in to help guide their students through problems toward a single answer and using a single approach (Stigler & Hiebert, 2009).

The types of problems used in MTCs are what Boaler (2014) calls "low-floor, high-ceiling" problems. Such problems are easily accessible to all and, yet, could also be extended to employ more sophisticated approaches. They are markedly different than the artificial problems commonly used in school mathematics, which confound students because of their artificiality (Gerofsky, 2004; Wiliam, 1997). Solving "low-floor, high-ceiling" problems requires probing, investigative work. The Brownie Problem is one such problem; here are several others:

> **The *Futurama* Theorem.** Professor Farnsworth and Amy build a machine that allows them to switch minds. They use the machine, so that Farnsworth can feel youthful in Amy's body, and so that Amy can eat whatever she wants, guilt-free, in Farnsworth's body. After using it, they discover that the machine has one, troublesome flaw: after a pair of minds is switched, the same two minds cannot switch back to their original bodies. What are Farnsworth and Amy to do?
>
> **The Key Cap Problem.** Rushing to get inside your house, you drop your key ring on the ground. What is the smallest number of colored key caps that you need to distinguish all of your otherwise indistinguishable keys?
>
> **The Four and Five Problem.** Using the numbers 1, 2, 3, and 4 no more than once and the operations of addition, multiplication, and subtraction, what are all possible results that you could obtain? What's the smallest positive whole number that you *cannot*? What about with 1, 2, 3, 4, and 5?

If you study these problems carefully, you will note that they are unlike traditional math problems. In most, there is no clear-cut statement of a "problem." Instead, a particular situation is presented from which new questions emerge. For instance, with regard to the Key Cap Problem:

- Should we agree that all the keys are physically indistinguishable?
- Is the key ring marked in some way?
- How many keys are on the ring? Does this even matter?

The Key Cap Problem is deceptively simple until the above questions are considered. In order for an actual "problem" to emerge, in the sense of an unknown or unknowable question, we assume that the keys are indistinguishable and their number unknown, and that the ring is unmarked. That said, there are additional variations that are also worth exploring, and in the PAMTC we would mention these before settling on one, agreed-upon problem. Much of our initial work involves simply interrogating what the problems *could* mean. This implies, of course, that our problems rarely have a single correct answer, nor a single solution path. This is one essential aspect of a mathematician's work: precisely identifying a particular question that is worth investigating, defining the important terms, and agreeing upon definitions.

Invariably, we get stuck when solving problems. More often than not, the stickiness involves difficulty articulating or justifying our ideas. When this happens in the PAMTC, we arrive at a teachable moment: this is what our K-12 students experience daily, when they face new mathematical ideas. The essence of mathematical inquiry is not feeling defeated, but rather, recognizing that getting stuck is when actual learning can occur. (The Common Core Practice Standards thereby value "persisting in problem-solving" [NGA Center / CCSSO, 2010, MP1].) Getting an answer, so to speak, is less important than developing strategies for how to become unstuck, such as making a table to organize information, working backwards, and trying a simpler case. These strategies invariably allow for progress; we then celebrate the joy in surmounting a block.

Note that in my description of MTC workshops, I have distinguished between "mathematician" and "professional mathematician"; this distinction reflects our belief that, fundamentally, *everyone is a mathematician.* Sure, those who make a living doing mathematical research have more experience than we do—those folks are "professional mathematicians." But we emphasize that pursuing and enjoying mathematics should not be exclusively reserved for professionals, certainly not any more than photography, music, or writing should be. With regard to writing, Atwell (1987) asserts:

> Teachers who write as a way of making their own meanings know about disorder….When we make room for the tentativeness and turbulence of creating written meaning, we and our kids breathe a collective sigh of relief. Writing well isn't a gift God gives to a chosen few. Instead, we provide a powerful demonstration: with enough time to shape and reshape the writing, with topics and audiences we care about and with responses along the way, anyone can write well. (p. 154)

Replacing "writing" with "doing mathematics," Atwell's words are equally apt descriptors of the *everyone-is-a-mathematician* philosophy underlying MTCs.

Again, doing math involves making discoveries and justifying these discoveries, rather than simply following someone else's prescribed rules without knowing exactly why. Some discoveries might be personal discoveries, but they are still important. Some discoveries may even have relevance beyond our workshops. For example, middle and high school teachers in the Palo Alto, CA MTC recently blurred the lines between professional and amateur mathematicians by publishing research in *The College Mathematics Journal* (Baker et al., 2013). Asked about his MTC team's work, mathematician-facilitator Brian Conrey said, "There are plenty of good, unsolved problems *that anyone can work on*. You don't necessarily need specialized training in mathematical research, so much as a willingness to try something and to be persistent" (Peterson, 2013, italics added).

A New Flavor of MTC: The PAMTC

At each of our workshops in the PAMTC, we draw upon the AIM model, but we differ in our approach in several significant ways. First, we devote a considerable amount of time to making the AIM model transparent to our participants. At the beginning of our workshops, we explain our purposes for coming together: to put ourselves into the position of our students, to become math learners for the next few hours, and to reflect upon our teaching. We announce that we will work together in solving a complex math problem. We stress the importance of productive collaboration and of asking each other questions. We say that it's OK to feel frustrated. After all, problems are (by definition!) intentionally frustrating situations. We note that growth only happens when you learn how to handle and work through challenges. We also say that it's OK to ask for more time to think. We encourage everyone to just try something when encountering an obstacle: make a table, write a list, draw a sketch. We explain that, by participating actively and having fun, you are doing math and building your problem-solving toolkits. And we say that we hope our participants will, in turn, share this understanding and these tools with *their* students, whenever *they* view math in narrow ways or become stuck when solving problems.

Secondly, in the PAMTC, we believe it's not enough to just do math. Teachers should also have opportunities to discuss how to create math-positive environments and foster authentic mathematical experiences in their classrooms. Therefore, beyond simply modeling a more engaging form of mathematical pedagogy—one centered on rich problem-solving experiences, discussions, and debates—we also debrief after each session, to bring the underlying principles to light. Our model for debriefing contains two pedagogical ingredients: how to make connections to curriculum materials and standards, and how to facilitate classroom inquiry. With regard to making connections to curriculum and standards, we analyze material from commonly used teachers' guides or standards documents, looking at the language and suggested activities, as well as discussing ways to understand and support students' learning. With regard to facilitating classroom inquiry, we look at videotapes of teaching, and we discuss strategies for scaffolding and differentiating problems, promoting student-thinking, and navigating classroom conversations. Some of these strategies are research based, and some emerge organically from our participants.

In general, we aim to support teachers in building the capacity to do problem-solving in their classrooms and to feel more comfortable letting their students do more of the intellectual work. One key challenge involves managing students who are not accustomed to doing so. Authentic problem-solving experiences place teachers in the role of facilitator rather than teller—that is, teachers must engage students in solving problems on their own and in talking with each other, rather than simply explaining what needs to be done. This is often a new role for teachers, and making such a shift requires a lot of deliberate thought and retraining. Instead of giving teachers lists of tips or activities to bring back to their classrooms, we try to build teaching capacities (akin to the aphorism of teaching a person *to fish*, rather than giving *a* fish).

Conclusion

In the PAMTC, such a community is building, as the majority of our attendees attend several workshops per year and our attendance continues to grow. Teachers rave about our work together, too. They report a deepening appreciation for mathematics as a field of creative inquiry and greater confidence in facilitating open-ended, problem-solving lessons in their classrooms. Most importantly, our participating teachers tell us they are having fun *doing* math!

I've argued that doing mathematics is much more than just grading homework or completing workbook exercises. Instead, through the Brownie Problem, I have illustrated that *really* doing math is like wrestling with a brain-teaser or creating a new dish. Real mathematics involves imagination, questioning, testing, and exploring. Furthermore, authentic, deep-thinking problems permit a variety of interpretations and approaches toward a solution.

We want our students to be able to do *real* math: to become creative problem-solvers and critical thinkers, to be engaged in inquiry, and to have a thorough grasp of concepts. We need them to understand the *whys*, and not just the *whats* and *hows*. To do so, they need to engage in actual mathematical inquiry. Borrowing a sports metaphor, students need to get in the game and play it as it is meant to be played—not just do endless practice drills (or worksheets). They need to get into a real kitchen and invent something new, not just play with an Easy Bake Oven. They need to write, themselves, and not just read what others have written.

The same should be said of our teachers. We want our teachers to be able to inspire our students, but since their own experiences with math were probably uninspiring, they need a different kind of professional development. Professional learning experiences cannot replicate outdated pedagogy, and professional learning—to be truly transformative—must be ongoing and lifelong. As there are very few resources and little time within today's schools (especially in under-supported districts like Philadelphia), professional learning experiences need to be more readily integrated with teachers' out-of-school intellectual and social lives. Most importantly, like writing a poem, creating a painting, or baking a dessert, PD needs to be fun and active. Like being in a book group, or attending a dinner party, it also needs to be a meaningful community experience.

Since 2006, MTCs have challenged the impoverished, bland, common perceptions of mathematics—that solving a math problem simply involves following a prescribed set of steps and practicing these steps in an unthinking way, over and over. For the past three years in Philadelphia, the PAMTC has also supported teachers in connecting their growing mathematical appreciation and understanding to more effective pedagogical practices. It may be a vain, budding notion, but it is nonetheless our hope that when asked what we do in our free time, we can proudly respond, “We discover mathematics”—and our students, their family members, and our colleagues will all want to join us.

Acknowledgements

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About the Author

Josh Taton is a PhD student in Teaching, Learning, and Leadership at the University of Pennsylvania Graduate School of Education. He earned a bachelor’s degree in mathematics from Yale University, and he worked as an actuarial consultant and a teacher before coming to Penn. He really does have a lot of outside interests, and he really loves discovering math, too! He invites you to visit, join, or support the PAMTC, and he may be reached at jtaton@upenn.edu.

ATMOPAV Invites Requests for Innovative Mini-Grants

. . . for creative and diverse projects which are designed to enhance and enrich the teaching of mathematics. Grant funds are available to be awarded annually in each of the three levels of elementary, middle, and high school. Grant requests for up to $1000 per year are welcome. The Innovative Grants are awarded to teachers to support creativity in the teaching of mathematics. To enhance this creativity, ATMOPAV encourages teacher collaboration on projects. Teams of teachers may apply. Tap into the energy of others and develop synergy among your colleagues. Talk to each other! The process for application is simple, and the decision process is efficient and streamlined. In order to find detailed information and apply, you can access the form at the ATMOPAV website: www.atmopav.com

Other ATMOPAV Awards:

Information about the *Mabel Elliott Student Teacher Award* and *Alan Barson Novice Teacher Award* can be found on our website. Help ATMOPAV support excellence in mathematics education by nominating people for these major recognitions. These awards are named in honor of two long-time ATMOPAV members whose leadership and dedication made a significant contribution to mathematics education in the 5-county Philadelphia metropolitan area. Someone you nominate may be our next iconic leader. Look around, and give us a name or two, please.

Local Teacher Receives Award from MET

The Mathematics Education Trust Board of Trustees recently awarded to Monique A. Carter, Widener Partnership Charter School, Chester the *Engaging Students in Learning Mathematics for Grades 6-8 Grant*, which is supported by the Veryl Schult-Ellen Hocking Fund. This award is to recognize teachers who incorporate or create middle school classroom materials and/or lessons that actively engage students in tasks and experiences to deepen and connect their current knowledge. The MET Board of Trustees congratulates Ms. Carter and the Widener Partnership Charter School for her successful proposal. The next deadline for applications for MET Grants is May 4, 2015. Please check the website at www.nctm.org/grants/ for information on all grants, awards and scholarships.

> “I know that 2 and 2 make 4 . . . though I must say if by any sort of process
> I could convert 2 and 2 into 5, it would give me much greater pleasure.”
> — George Gordon Byron

Results of MATHCOUNTS 2015 Competitions

Pennsylvania State MATHCOUNTS Competition – March 28, 2015

Individual Student Results (Top Ten Students)

Place	Student	Chapter/County	School
1*	Christopher Lee	Pittsburgh Chapter	Carson Middle School
2*	Evan Qiang	Montgomery Co.	Wissahickon Middle School
3*	Aaron Li	Central Chapter	Park Forest Middle School
4*	Baron Cao	Central Chapter	Park Forest Middle School
5	Daniel Li	Lehigh Valley	Nitschmann Middle School
6	Kevin Zhou	Montgomery Co.	Arcola Intermediate School
7	Andrew Huang	Chester Co.	Tredyffrin Easttown Middle Sch.
8	Kiran Rebholz	Montgomery Co.	The Shipley School
9	Kevin Wu	Chester Co.	Tredyffrin Easttown Middle Sch.
10	William Huang	Central Chapter	Park Forest Middle School

*The top four students will represent Pennsylvania in the 2015 Raytheon MATHCOUNTS National Championship in Boston, MA, in May, 2015. (Alternate student is #5.)

Special Awards (Written Competition)

	Individual	Chapter/County	School
Highest Scoring Female:	Sai Mamidala	Delaware Co.	Garnet Valley MS
Highest Scoring Male:	Christopher Lee	Pittsburgh Ch.	Carson MS
Highest Scoring in Grade 7:	Evan Qiang	Montgomery Co.	Wissahickon MS
Highest Scoring in Grade 6:	Daniel Li	Lehigh Co.	Nitschmann MS

Top Three (School) Pennsylvania Winning Teams

Place	School Team	Chapter/County
1**	Park Forest MS	Central Chapter
2	Tredyffrin Easttown MS	Chester County
3	Fort Couch MS	Pittsburgh Chapter

** Park Forest MS coaches Rebekah Sjoberg and Susan Owens will coach PA team in National Championship.

Winners of County MATHCOUNTS Competitions (5 Counties) – Feb., 2015

Individual Students

Bucks Co.: Rishi Mago (Newtown MS); Frederick Qiu (Holicong MS); Jasmine Zhang (Newtown MS); Margaret Zheng (Newtown MS)

Chester Co.: Gokul Murugadoss (Lionville MS); David Wang (Tredyffrin Easttown MS); Runya Xu (Valley Forge MS); Kenny Yang (Great Valley MS)

Delaware Co.: Priya Ganesh (Radnor MS); Sai Mamidala (Garnet Valley MS); Joshua Yoo (Radnor MS); Daniel Zhu (Garnet Valley MS)

Montgomery Co.: Evan Qiang (Wissahickon MS); Kiran Rebholz (The Shipley S.); Ryan Zhao (Wissahickon MS); Kevin Zhou (Arcola Intermediate S)

Philadelphia: Calvin Hu (J. R. Masterman S.); Malana Li (Germantown Friends S.); Jonah Taranta (Friends Select S.); Anthony Moore (The Philadelphia S.)

School Winning Teams (5 Counties) – Feb., 2015

Bucks: Newtown MS **Chester:** Tredyffrin Easttown MS **Delaware**: The Episcopal Academy
Montgomery: The Shipley School **Philadelphia**.: J.R. Masterman School

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Our Newsletter Printer Submits An Interesting Link

What can I say about the wide-ranging appeal of mathematics, despite evidence to the contrary? Our wonderful current printer and distributor of this newsletter (who is based on Cape Cod) reads every issue we send him and discusses some of its content with his employees and family. Isn't that really cool? Somebody did right by him in junior high. Anyway, our printer/guru Tom Spollen sent me this email back in February:

"Linda and I saw an unusual screening of *The Imitation Game* a week ago in Chatham. The local Chatham Marconi Maritime Museum sent a representative to the theater with an actual Enigma machine and there was great interest. I sent some follow-up questions about American cryptology to the docent and received the attached as part of his response. I'm sure several ATMOPAVians would find this interesting."

The link to the pdf document to which Tom refers is here:
https://www.nsa.gov/about/_files/cryptologic_heritage/publications/wwii/solving_enigma.pdf

Some of you probably have students as well as colleagues who would enjoy reading it. Thanks, Tom! -- LH

FEMTO – A simple card game with a lot of depth

The NRICH website (www.nrich.maths.org/frontpage) is based at the University of Cambridge in the UK. There is a wealth of material there for every level of mathematics. Here is a delightful game created by one of its supporters, Alan Parr, followed by some of his musings on how it might be modified. It could be an interesting exploration as your year is winding down.

Rules for FEMTO

1. Femto is a card game for two players; a Femto pack consists of eight cards numbered 2,3,4,5,6,7,8,10.
2. The cards are shuffled and dealt, so each player gets four cards.
3. In each round of play each player puts out one card, face down. The two cards are then turned face up.
4. The round is won by the higher value card, unless the higher card is more than twice the value of the lower, in which case the lower card wins. e.g. 10 beats 8, 6 beats 5, 3 beats 10, 10 beats 5, ...
5. Whoever played the winning card chooses one of the two cards and puts it, face up, on the table in front of him/her. The player of the losing card takes the remaining card and puts it back into his/her hand.
6. More rounds are played until one player has no cards left.
7. The winner is the player with the greater total value of cards in front of them at the end of the hand.

l For a tiny game there seem to be plenty of questions to explore. How long do games last? What is a typical winning score? How many different hands are possible? Which cards are the most powerful? Indeed, what is the most powerful hand (and how could it be beaten)?

Then there's another set of questions, and as a game developer these are the ones which particularly interest me. The first version of any game is often interesting enough but needs a bit of tinkering to turn it into the finished version. Perhaps the most obvious question to ask is whether we've got the right set of cards. To take another set almost at random, would 5,7,9,13,16,18,20,24 be any better? Or perhaps we should stick with the original set but add a 12: deal out four cards each so that one card is left unused. Or perhaps there's a role for a Joker card.

Other questions which I'd want to explore concern the game structure. I've suggested that the cards are dealt out, but what if the players take turns to choose cards to make up their starting hand? And instead of playing simultaneously, what if each round is played with one player leading a card face up, so that the second player can see its value before responding? I've also got a hankering to change rule (5), so that it's the losing player who chooses which card goes back into his/her hand. Or perhaps it should be not the winner or loser of the individual round who chooses, but whoever is currently winning (or perhaps losing), who makes the choice . . .

Changing the overall winning criterion often has interesting consequences. The most usual way to do this is to make the winner the person with the lower score rather than the higher. Or in this case, the aim might be to score an odd total, or a total that is a multiple of 3, or a prime number . . .

To read more about Femto and its possibilities, go to the NRICH website.

FROM NCTM . . .

If you haven't been to the NCTM website recently, you might be surprised. You'll find web-based games, wonderful puzzles and problems, and much more. Here is a sampling. You'll find more about each one (and the solutions) at their site.

Triangular Number Order

Ten is a triangular number, because 10 objects can be neatly arranged in an equilateral triangle.

In general, a triangular number is a number that can be represented as a triangle with one object in the first row and each subsequent row contains one more element than the previous row. (From the picture above, you might notice that 1, 3, and 6 are also triangular numbers.)

Order the digits 1 through 9 so that the sum of any two adjacent numbers is a triangular number.

Blocks Cubes

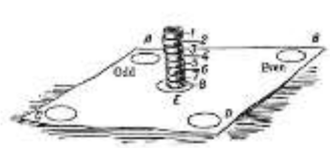
A rectangular wooden block (not necessarily a cube) is painted on the outside and then divided into one-unit cubes. As it happens, the total number of painted faces equals the total number of unpainted faces. What were the dimensions of the block before it was painted?

Clock Squares

Starting at 12:00 midnight, you wait a number of minutes that is a perfect square and then look at a digital clock. The number you see (with the colon removed) is also a perfect square.

What is the first time after midnight that this happens?

Even and Odd



Number 8 checkers and pile them up as shown. Use a minimum of moves to shift checkers 1, 3, 5, and 7 from the center to the "Odd" side circles and checkers 2, 4, 6, and 8 to the "even" side circles. To move, shift the top checker from one pile to the top of another. It is against the rules to put a checker with a higher number on a checker with a lower one, or to place an odd-numbered check on an even-numbered checker or vice versa.

Thus, you can put checker 1 on 3, 3 on 7, or 2 on 6 – not 3 on 1 or 1 on 2.