ATMOPAV NEWSLETTER

PDF edition

Winter 2014

Association of Teachers of Mathematics Of Philadelphia And Vicinity

ATMOPAV

An affiliate of NCTM & PCTM

Keep these dates:

ATMOPAV Spring Conference and Banquet ~ Germantown Academy ~ March 20
PCTM Conference ~ Hershey ~ November 5 - 7
NCTM Annual Meeting & Conference ~ New Orleans ~ April 9 – 12
NCTM Regional Conference ~ Indianapolis, IN ~ October 29 - 31
NCTM Regional Conference ~ Richmond, VA ~ November 12 – 14

In this issue:

Winter 2014

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ATMOPAV

NEWSLETTER

The Association of Teachers of Mathematics of Philadelphia and Vicinity

An Affiliate of the National Council of Teachers of Mathematics and the Pennsylvania Council of Teachers of Mathematics

President's Message

Susan Negro
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On December 7, 2013, I was perusing the *New York Times* when I came across the piece, "*Who Says Math Has to be Boring*?" posted by the NYT Editorial Board. Anthony Carnevale, director of the Center on Education and the Workforce at Georgetown University reports that, "Only 11 percent of the jobs in the STEM fields require high-level math, but the rest still require skills in critical thinking that most high school students aren't getting in the long march to calculus." Carnevale's pontifications and the article caused quite a racket on social media. Educators, parents, employers and students weighed in on the suggested changes that would make the study of mathematics exciting, and therefore "less boring" for students.The Editorial Board touts a more flexible curriculum in conjunction with very early exposure to numbers as a means to ensure that we provide a pathway to critical thinking and problem solving. The traditional math sequence hasn't changed for many generations: arithmetic, pre-algebra, algebra, geometry, trigonometry, and calculus. The article posits that deviations from this classical pathway may be viewed suspiciously by most high schools throughout the United States; however, calls for change in the high school curriculum to include engineering, computer science, and other technical courses have been made by the US Secretary Arne Duncan and President Obama. But will these changes to a more technical curriculum make math less boring and more entertaining?

Karsten Stueber, a professor of philosophy at the College of the Holy Cross, says "We do not do our children any favors by telling them that math is fun. Such a view encounters what philosophers call the paradox of hedonism: the more one tries to have fun, the less likely one will have it. Skills must be mastered. However, mastery can be a lot of fun." The question then becomes when do we introduce our youngest students to the mathematic skills that must be mastered? According to a 2013 long term study conducted by researchers at the University of Missouri, "Children who don't grasp the meaning and function of numerals before they enter first grade fall behind their peers in math achievement, and most of them don't catch up." Chris Cain, a mathematics consultant to the US Department of Education, echoes Stueber's call for a more tradition approach: "Memorizing multiplication tables isn't a bad thing, and sustained emphasis in the US should be made to build children's numeracy skills as early as possible."

Additionally, poor teacher preparation was blamed for "boring math" classes by the Editorial Board. Carnevale reports, "What we're talking about nowadays is an integration of career and tech education. It takes money and talent to make that happen. It requires a different kind of teacher." Like many of the Carnevale's critics on social media, I counter with the two factors that contribute to what good teachers say and how they say it with Solomon Friedberg's requirements: **experience and good judgment**. You may leave talent in, but take money out of the equation, please! Our contemporaries ask the important questions every day: Why is this mathematical concept important? How do we balance conceptual understanding and an appreciation of the big picture with technical details and problem solving? What will motivate the students and engage them intellectually?

As any good editorial board will, the NYT tracks and reports on trends. However, what we do not need in mathematics is another "fundamentally different approach." Remember the "new math" of the sixties? Instead, we should heed the wise words of Hung-Hsi Wu, emeritus professor at the University of California-Berkley, who has long advocated for renewed attention to teaching teachers the basics so that they can pass them on to students.

Let's not leave the essential variable out of Carnevale's reform equation: the natural curiosity of children. Math class is a boundless place to exploit this evolutionary trait and innovative teachers are doing this daily. Educational leaders in communities across the US are creating project based curricula and national catalogues of remarkable, engaging projects ripe for the plucking. Take a minute or two on one of our snow days this winter and check out the work of math educators Margaret Schwan Smith, Mary Kay Stein, and cognitive scientist Dan Willingham. I think you will be inspired!

The ATMOPAV Newsletter is published three times a year. Contributions in the form of articles, reviews, teaching ideas, humor, and opinionated essays are welcome. Material for the Spring 2014 issue should be submitted no later than March 15, 2014. Please be sure to include contact information (e-mail and/or telephone) if you are not a regular columnist. Send your text to the editor:

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The Association of Teachers of Mathematics of Philadelphia and Vicinity was founded more than 60 years ago. It is an organization of mathematics educators who are dedicated to improving mathematics instruction at every level from kindergarten to college. Through its professional meetings, website, and this publication, ATMOPAV supports, informs, and facilitates communication among teachers, pre-service education students, supervisors, and school administrators from the five-county Philadelphia area. Our awards program recognizes excellence among student teachers, novice teachers, and secondary school students who participate in our annual mathematics competition.

To join ATMOPAV, please
complete the membership application in this issue.

Call for Columnists

If you're an educator, you have something to share, and this is a good place to do it. We are looking for more writers of regular columns and of single articles. About what, you ask? Consider these possibilities:

- Activities, especially for grades K - 5
- Mathematics software reviews
- Adapting lessons for students with special needs
- Cross-curricular activities that link math with other subjects
- Diary of a first-year teacher
- Recollections of a retired teacher
- Mathematics-related history
- Useful resource books for teachers
- Math games and puzzles (esp. for elementary and middle school)
- Literature connections and book reviews
- Reviews of math-related apps
- Teaching with particular software: Scratch, SketchUp, spreadsheets, Geometer's Sketchpad, etc.
- Working with challenged learners
- Parent Night/ Math Night ideas

. . . *and whatever else you might think of.*

We are eager for material that applies to any level of mathematics education. If you don't want to write but know a colleague who may, please pass the word. Contact the editor for further information.

Math-Related Websites for Teachers and Students

Cut the Knot http://cut-the-knot.org ***The Futures Channel*** http://thefutureschannel.com
Making Mathematics http://www2.edc.org/makingmath/default.asp
Don's Math Website http://www.shout.net/~mathman/ ***CoolMath4Kids*** http://www.coolmath4kids.com
JETS website http://www.jets.org/latestnews/JETS_Challenge.cfm
The Virtual Library of Interactive Mathematics http://www.matti.usu.edu/nlvm/nav/vlibrary.html
The Intermath Dictionary http://jwilson.coe.uga.edu/interMath/MainInterMath/Dictionary/welcome/howto.html
Mathematrix http://www.geocities.com/CapeCanaveral/Hangar/7773/
The MacTutor History of Mathematics http://www-groups.dcs.st-and.ac.uk/~history/
Powers of Ten http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html
Mathworld http://mathworld.wolfram.com/ ***RadicalMath*** http://www.radicalmath.org
S.O.S. Mathematics http://www.sosmath.com/ ***Natural Math*** http://www.naturalmath.com/
The Knot Plot Site http://www.cs.ubc.ca/nest/imager/contributions/scharein/KnotPlot.html
Paper Models of Polyhedra http://www.korthalsaltes.com/index.html
Mrs. Glosser's Math Goodies http://www.mathgoodies.com ***Platonic Realms*** http://www.mathacademy.com
The Math Forum@Drexel http://www.mathforum.org/ ***Explore Learning*** http://www.explorelearning.com
Khan Academy http://www.khanacademy.org/ ***XtraMath*** www.xtramath.org
And, of course . . . ***The National Council of Teachers of Mathematics*** http://www.nctm.org/

Little Folks, Big Ideas

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The Miquon School

An Enjoyable Number-Sense Activity

In my first and second grade blended classroom at the Miquon School, we often will do the following number sense activity, especially at the beginning of the school year. However, the activity becomes more rewarding for the children as the year goes on, mainly because the children discover just how much imagination they can put into it. It is really quite simple – all you will need is a piece of chart paper and a marker, although chalk and blackboard work great, too. The advantage to chart paper is that you can keep it to revisit with students later in the year and let them see how their ideas have grown.

First, choose a number to work on. Any number will do, and there are a lot of them -- actually, an infinite quantity. Next, write the number nice and big on the chart. Then ask the children to think about the number and how they might show it, and what it means to them. Be sure to ask them to just *think* about it; calling out ideas will not do for now. Then, ask the children to talk to a child next to them about the number. After about a minute of chatting, ask for some ideas from the children. Write and/or draw what comes from the discussion and talk about it. Simple as that!

Being with the younger set, I usually pick 5 or 10 as our first number. So after drawing the 10 nice and big, allowing time for the children to think and to talk with their neighbor, I write and draw their smart ideas. In the beginning of the year, things seem pretty slow. Ideas are not flowing so quickly, and sometimes it is a struggle to pull some ideas out, but not to fear -- it will happen. It might start with 10 pieces of candy, a dime, ten fingers or toes, the word ten, or things like that. But as I ask more questions to foster some ideas to think about, imaginations really do kick in. One child mentioned that for 10 it was the same as a bundle from our place value chart. Other interesting number sense ideas included for the number, 14, “Oh, that’s two weeks.” We also heard that 14 was the letter N in the alphabet. Another child said for 20, “We can use two dimes for 20.” And another child added, “And that could be four nickels, too.” The direction of where this activity can go is limited only by the children’s imagination.

What has been valuable about this activity is that the children are putting meaning to numbers and how they can be used in real life. The think time allows all of the children an opportunity to formulate an idea. Also, the discussion with a neighbor allows children to hear some other ideas and maybe decide that theirs is a good idea, one that they would like to share. By writing and drawing the children’s ideas on the chart, we are showing that their ideas are valued and becoming a part of our classroom learning. This is a quick and simple activity that has a lot of benefits, and I would highly recommend returning to it all year long.

MIDPOINT

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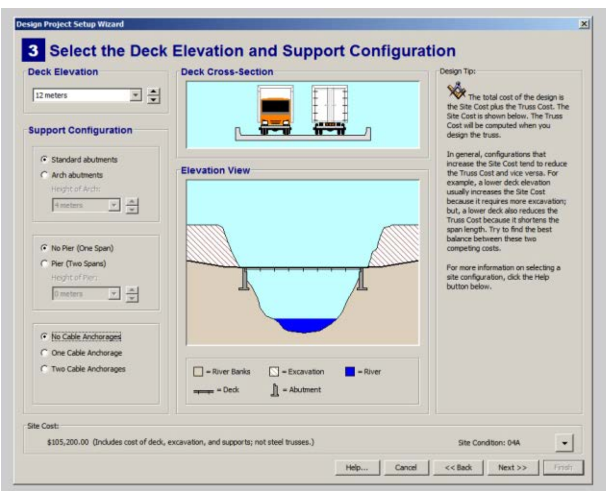
Bridge Work (and Play)

Did you know that the military academy at West Point was the first college in the USA to offer formal instruction in engineering? A delightful descendant of that early 19th-century curriculum is the *West Point Bridge Designer* software, which is available free to anyone and is also used in a competition for 6th grade and up which is now sponsored by the American Society of Civil Engineers and The Army Educational Outreach Program. If you want to know more about the competition or just want to obtain the software, go to www.bridgecontest.org and follow the links. It's available for PC and Mac – a true STEM learning environment that's been around a lot longer than the term itself. It's so rich in opportunities for applying many kinds of mathematics that this article can provide only a few of them. (Note: This article was written using the 2013 version of the software, as the 2014 version was not going to be available until mid-January. If you used this software some years ago and haven't looked at it since, you'll be pleasantly surprised by the changes.)

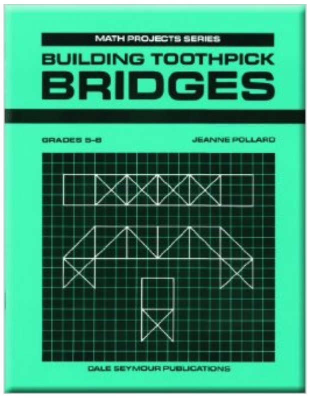
The goal is to build a strong two-lane bridge as cheaply as possible, choosing either a truss bridge or combination truss and suspension structure. The bridge needs to be able to support its own mass as well as that of a loaded truck that will drive across the bridge. The changing load and type of stress on each component as the truck crosses is illustrated by a clever animation that can be run over and over, using colors to show which members are under compression or tension. There is an entertaining Bill Nye ("The Science Guy") video about structures that makes those two stresses quite easy to understand, and it's available via YouTube.



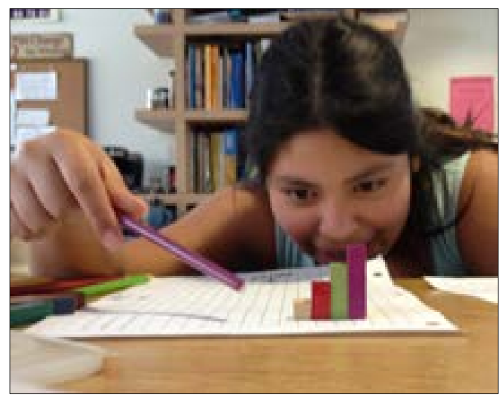
The bridge site is a valley with a river at the bottom. You can excavate the banks or leave the terrain as it is. The advice on the heavily-annotated screens is that increased site preparation costs will generally reduce the cost of the materials that make the bridge. This might lead to some interesting comparison graphs of the cost to build a longer bridge at the top of the valley or a shorter similar bridge with some excavation to lower the roadbed placement. The software keeps track of the total price of the bridge as it is built. Clicking on the calculator next to the total on the design screen will open a detailed list showing the things that contribute to that total.

Students can work entirely on their own or use a template to guide the shape of their bridge – either a "through" truss which is above the roadbed or a "deck" truss which supports the bridge from below. Beginners will benefit from starting with a template. Either way, they need to make decisions about the actual truss beams – the type of steel, the size of the beam in cross-section, and so on. When the bridge is stable enough to be tested, they can send the truck across and see if their design is successful. If they try to test an unstable bridge, a pop-up window will show them where the problems are. If the bridge succeeds in terms of carrying the load, students can then try to reduce the cost of their bridge by changing the design and/or the materials. There are unlimited undo edits, and multiple iterations of the same bridge can be saved. The finished bridge design is printable, as is the cost calculations report. The amount of data available as the bridge is constructed and tested is enormous. Students can see their bridge components analyzed piece by piece for dimensions, materials, cost, and ability to withstand tension and compression forces. It will almost certainly require teacher guidance to get most students to use this data effectively, but the time will be well spent.

Many students will choose to explore this software in their free time, often going outside of its intended purpose: making the most expensive bridge they can construct, for example, or delighting in having the bridge fail as the truck tries to cross. All of their playful exploration will add to their understanding of structures, and it often generates peer interest in ways that the assigned goals may not.



In my classroom, I use it in conjunction with building toothpick bridges and doing a cross-curricular research project on real bridges around the world. Students work in pairs to build their toothpick bridges. The original idea came from a book by Jeanne Pollard, which is still in print. The major changes I've made include having students work in pairs instead of in teams of 5 as the book suggests because there isn't enough for that many students to do throughout the project time, and we do it as a non-competitive activity that requires the bridge to support a target mass of 2 kilograms instead of finding the strongest bridge by stressing them all to the point of failure. The book, however, presents a well-conceived approach: students must measure distances and angles carefully to prepare their site (a piece of foam-core board), create a full-size design on centimeter grid paper that serves as a pattern for their bridge, and work within a budget that limits the number of toothpicks they can use. The inherent stability of triangles quickly becomes apparent.



A third component of our bridge-building unit is a few classes spent with activities taken from *Spatial Problem Solving With Cuisenaire Rods* by Patricia Davidson, which is no longer in print, but used copies are still available. The section that has students identify and build rod structures from front, top, and side views is a good way to introduce drawing their toothpick bridge plans from the same perspectives.

The bridge software site mentioned at the start of this article also has a detailed section on building truss bridges from file folders. It's listed under "resources" at the top of the home pages and contains extensive downloadable pdf lesson plans. These bridges require a lot of patient, precise cutting and folding of the light card stock; even their simplified bridge plan is challenging to build correctly.

For younger students, a better resource for paper bridges is the classic work by Mario Salvadori, *Strength Through Form: Paper Bridges*. This and many other teaching resources focusing on urban architecture and engineering are available directly from the store at the Salvadori Center: http://www.salvadoricenter.org/.

There are a lot of websites and videos available to add information and spark interest in this entire topic. Here are just a few:

http://www.pisymphony.com/toothpick/toothpick1.htm
http://www.mrg-online.com/bridge.htm
http://www.instructables.com/id/Toothpick-Bridge-Project/
http://www.youtube.com/watch?v=0wyDIT1xNQ4
http://lasd.k12.pa.us/teachers/purnellj/bridgeinst.html
http://www.garrettsbridges.com/

Have fun!

TIMEWASTERS

Don Scheuer
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The fall has come and gone and we now languish in the doldrums of winter. Golf courses are snow-covered or muddy, and the wind chill is more of a challenge to our wind shirts. Other than the special holidays, this time of year is a bummer.

In the last column, I wrote that I was taking an auto trip around Pennsylvania to visit some historic sites, some national park sites, and some natural wonders of the state. Alas, the government shutdown rendered some of the destinations closed, so the trip was postponed until further notice, as they say. I'm looking forward to the reinstatement of the trip in the near future.

It seems that the government shutdown also affected our usually stable set of problem solvers, as well. No responses to the last set of puzzle gems were forthcoming, so we plug on with a new batch and hope for the best.

Here are the puzzles and solutions to the last offerings:

A. **On my trip around Pennsylvania, I will visit the town of Treasure, PA. Each year, the town holds a treasure hunt. Here are the directions given to the participants:**

Walk 108 paces west, 315 paces south, then return to your starting point walking in a northeasterly direction. Find the number of palindromic between 0 and the number of paces you walked. The treasure is behind the fence post that corresponds to that number. Behind which fence post is the treasure?

Solution: The Pythagorean Theorem can be used to find the number of paces walked in a northeasterly direction.

$a^2 + b^2 = c^2$. So, $108^2 + 315^2 = c^2$. Then $c = 333$.

To find the treasure, a participant must walk a total of 756 paces. Determining the number of palindromic numbers between 0 and 756 can be done using groups of number. From 0 to 99, there are 9 palindromes (11, 22, 33 … 99). From 100 to 199, there are 10 (101, 111, 121… 191). Since there are 6 groups of 100 from 100 to 699, there are 60 palindromes from 100 to 699. From 700 to 756, there are 5 palindromes (707, 717, 727, 737 and 747). So, the total number of palindromes from 0 to 756 is 74. The treasure is behind fencepost 74.

B. **On my trip around PA, I will take some old problems from the AHSME (American High School Mathematics Examinations) competitions to amuse me before bedtime. Suppose you took the 120 permutations of the letters in AHSME and arranged them in dictionary order, as if each were an ordinary five-letter word. What would the last letter of the 86th word be?**

Solution: The first 24 words (4!) begin with A. The next 24 begin with E and the next 24 begin with H. So the 86th word will begin with m and it is the 14th such word (86-72). The first six words which begin with m begin with MA and the next six with ME. So the desired word starts with MH and it is the second such word. The second word which would begin with MH is MHASE. Therefore E is the last letter.

C. **Back in the 1700's a group of 500 settlers arrived in Pennsylvania with provisions for 60 days. Twelve days later, another group of settlers joined them, but did not have any provisions with them, so both**

groups shared what remained of the original provisions. These provisions lasted only 40 days longer. Assuming each person consumed an equal amount each days, how many settlers were in the second group?

Solution: The original total was 30,000 settler-days of provisions (500 x 60). After 12 days, there would have been 24,000 settler-days of provisions remaining. If this lasted 40 days, there must have been 600 settlers consuming the provisions. So, the second group had 100 settlers.

And a partridge and a par three.

Here's the next batch of conundrums . . .

A. A circle has its center at (2, -3). The equation of a tangent to the circle is $4y - 3x = 7$. Find the equation of the circle.

B. The name ROY G BIV is a mnemonic way to remember the principal colors of the spectrum: red, orange, yellow, green, blue, indigo and violet. Suppose there was a design consisting of three square spaces in a row and each of these spaces is to be filled with a different color, left to right, from this list. Furthermore, the colors must be in the same order as they are in the list. So, RBV would be allowable, but BGV and YGR would not be allowable. In how many ways can the spaces be colored?

C. In an arithmetic progression, the 25^{th} term is 2552 and the 52^{nd} term is 5279. Find the 79^{th} term.

See you when the dimpled spheroids are flying again!

Send solutions to Don at his email address.

ATMOPAV Invites Requests for Innovative Mini-Grants

. . . for creative and diverse projects which are designed to enhance and enrich the teaching of mathematics. Grant funds are available to be awarded annually in each of the three levels of elementary, middle, and high school. Grant requests for up to $500 per year are welcome. The Innovative Grants are awarded to teachers to support creativity in the teaching of mathematics. To enhance this creativity, ATMOPAV encourages teacher collaboration on projects. Teams of teachers may apply. Tap into the energy of others and develop synergy among your colleagues. Talk to each other! The process for application is simple, and the decision process is efficient and streamlined. In order to find detailed information and apply, you can access the form at the ATMOPAV website: www.atmopav.com

Other ATMOPAV Awards

Information about the *Mabel Elliott Student Teacher Award* and *Alan Barson Novice Teacher Award* can be found on our website. Help ATMOPAV support excellence in mathematics education by nominating people for these major recognitions. These awards are named in honor of two long-time ATMOPAV members whose leadership and dedication made a significant contribution to mathematics education in the 5-county Philadelphia metropolitan area. Someone you nominate may be our next iconic leader. Look around, and give us a name or two, please.

IT'S TIME TO MARK YOUR CALENDAR . . .

The ATMOPAV Spring Conference and Banquet will be held on March 20, 2014, at Germantown Academy. Conveniently located in Ft. Washington near Route 309 and the PA Turnpike, this promises to be a wonderful way to learn something new, enjoy an excellent meal, and honor some colleagues. Watch for a flyer in your mail, and invite a non-member friend or six to join you!

Pennsylvania Statistics Poster Competition – Call for Posters

The Pennsylvania Statistics Poster Competition is hosted by Saint Francis University and sponsored by mathematics teacher organizations across the state, including ATMOPAV. The competition offers cash prizes and certificates for first, second, third, and fourth place in each grade level category (K-3, 4-6, 7-9, and 10-12) along with certificates for honorable mentions. The winning posters from the Pennsylvania Competition are then submitted to the National Statistics Poster Competition.

Teachers from across Pennsylvania are encouraged to incorporate the Statistics Poster Competition into their classrooms. Collaborative efforts between subjects can be an exciting way to show how mathematics relates to other subjects such as health, geography, history, science, etc.

Online registration for the Call for Posters will be available in September 2013. All posters MUST be registered online and postmarked by midnight **on February 28, 2014**. Judging will take place in March and winners will be announced in April 2014.

General information along with the rules and guidelines for the Pennsylvania Statistics Poster Competition can be found at:

www.francis.edu/pa-statistics-poster-competition
or by emailing ScienceOutreach@francis.edu.

Pictures of winning posters from previous years are also available on the web site.

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Leadership in Mathematics Education

As a member of PCLM, you will be kept on the cutting-edge of state and national issues confronting math education. Through newsletters, conferences, and mailings, PCLM provides opportunities to exchange ideas with colleagues and to effect the changes needed to improve mathematics education in Pennsylvania.

To learn more, visit the website: www.pclm.org .

WORDS FROM NCTM . . .

Are We Obsessed with Assessment?

by NCTM President Linda M. Gojak
reprinted from NCTM *Summing Up*, November 4, 2013

Although I have not been a classroom teacher for the past 12 years, my work with teachers and my role as president of NCTM allow me to spend a great deal of time in schools and classrooms. I see firsthand the amount of time spent on testing students. I worry about the instructional time we lose when we spend so much time on testing. I worry about the pressure we put on students when so much of their time in schools is spent being tested. I worry that we have taken the joy out of teaching and learning in the name of accountability that can be determined "only" by giving more tests

Assessment is a vital part of teaching practice. At the beginning of a unit, a pretest includes questions to provide us with information on whether students have the prerequisite skills for the upcoming topic and tells us what they already know about the topic. Using the data from pretests, we plan instruction that will meet our students' needs. We quiz students throughout the unit to check their progress. At the end of the unit, we give a summative assessment to find out what students have learned and what they still need to work on. Assessment is appropriate when we use the data obtained through this sequence to plan instruction that ensures student success.

We do not need to depend on pencil-and-paper assessments. A variety of assessment strategies can provide teachers with the information they need about their students' progress. Worthwhile formative assessment can happen when a teacher asks probing questions and pays close attention to student responses. A project that students complete at the end of a unit can represent both their conceptual and procedural knowledge and tell what they have learned—often better than a pencil-and-paper task. The good news is that more types of assessment, designed and controlled by the teacher, are finding their way into the mathematics classroom.

However, with the advent of high-stakes testing, the number of assessments we give students has greatly increased and their purpose has changed. Test preparation takes place in some classrooms every day, often consuming as much as one-quarter of instructional time. We teach to the test because we are told to. We have regular, building-wide testing days throughout the year. Often conducted in the name of "Common Formative Assessments," these exercises can take the form of multiple-choice tests that are prepared by someone in the district. They may align with the pacing chart but often have little teacher input and provide teachers with little information about their students' deep understanding of a concept. We give students tests to take home to practice for the "real" test. We have become a test-driven society!

It is time to have serious discussions about the purpose of assessment and its impact on the mathematics education of our students. The obsession with test preparation is more detrimental than helpful in ensuring student success and student motivation. Teachers need to collaborate with one another and with administrators to determine a testing policy and plan that supports their students and informs their instruction. We must be the voice that speaks on behalf of our students, focusing on meaningful learning rather than the pressure to perform.

I am not suggesting that we lower our standards for students. I am not suggesting that we eliminate good assessments. I am suggesting that we reexamine our obsession with test preparation and standardized tests that take days of instructional time, and that we carefully consider the impact this has on our students and on our teaching. No effective teacher wants to stop high-quality instruction to prepare for a test; rather, we know that high-quality instruction is the best test preparation we can give students.

Algebra: Not 'If' but 'When'

by NCTM President Linda M. Gojak

reprinted from NCTM *Summing Up*, December 3, 2013

One of the questions I am frequently asked by teachers, parents, and reporters is, "When should students take algebra?"

Let's assume that we're talking about a college preparatory algebra 1 course. The content and instruction must be designed to develop both conceptual and procedural understanding. For students to be considered successful in first-year algebra, the expectation must be that reasoning and making sense will be priorities of both teaching and learning.

Algebra has often been referred to as a "gatekeeper" to higher learning—both in mathematics and in other fields. Research shows that students who complete a mathematics course beyond the level of algebra 2 are more than twice as likely to pursue and complete a postsecondary degree. Students who don't do well in algebra compromise their career options, especially in STEM fields. The question is no longer *if* students should take algebra but rather *when* students should take algebra.

As recently as 20 years ago, most students took algebra in the ninth grade. Students who showed exceptional talent in mathematics might be offered the opportunity to take it in the eighth grade. In many schools today, algebra in the eighth grade is the norm, and students identified by some predetermined standard can complete the course in seventh grade. Algebra courses are even stratified as "honors" algebra and "regular" algebra at both of these grade levels. The variation in course names leads one to wonder about the level of rigor.

One reason for the push to offer algebra earlier is the poor showing of students in the United States among comparable industrialized countries on international assessments. The belief held by many is that giving students earlier opportunities to complete algebra and take more advanced mathematics courses at the high school level will solve this problem. However, the issue is more complex than simply offering students the opportunity to take algebra earlier.

Requirements for taking algebra in the middle grades should be clear and must not be compromised. Successful completion of a rigorous algebra course requires students to have prerequisite mathematical understandings and skills as well as a work ethic that includes the tenacity to stick with a problem or concept until it makes sense and the willingness to spend more time on assignments and class work. Furthermore, a key characteristic of students who are successful in algebra, no matter when they take it, is a level of maturity that includes a readiness to understand abstract mathematical definitions, to work with abstract models and representations, and to understand and make connections among mathematical structures—and this readiness should extend to making abstract generalizations.

Students and parents should be fully aware of course expectations, consequences for not meeting the expectations, alternatives to the study of rigorous algebra in the middle school, and options for future mathematics work. Moving a struggling student out of a middle school algebra course not only has social implications for the student, but also affects his or her self-efficacy, which is very important for success in future mathematics courses.

I recall an assignment from my undergraduate work in which we applied the Fry readability formula to Margaret Mitchell's novel *Gone with the Wind.* I still remember my surprise to find out that this novel was determined to be at a sixth-grade reading level. I realize that this does not indicate that it is appropriate to assign *Gone with the Wind* to sixth graders. It has been a while since I completed that assignment, but I often think about it when the discussion about accelerating students in mathematics arises. Just because a student can read the sentences in *Gone with the Wind* doesn't mean that she has the experience or maturity to deeply understand what she is reading. The same is true in mathematics. Just because a student can mimic steps shown by the teacher doesn't ensure that he has the sophistication to deeply understand the mathematics.

So, when should students take algebra? Many students and parents interpret taking algebra in the seventh or eighth grade as an indication of a level of superior intelligence — a status symbol. My experience, both as a student and as a teacher, leads me to believe that we do more harm than good by placing students in a formal algebra course before they are ready, and few students are truly ready to understand the important concepts of algebra before eighth grade. Many students should wait until ninth grade.

That does not mean that the middle-grades mathematics experience can't be rich or worthwhile—even beneficial and indispensable to students' future success in mathematics. I have always believed that middle school should be a time for students to get "messy" with mathematics. Students enter the middle grades with enough mathematical knowledge to explore mathematics through experiences that they may never have in high school or college. Seeing the relevance of mathematics in real-world situations and future career options encourages students to take more mathematics rather than to wonder, "When are we ever going to use this?" Solving interesting problems with high cognitive demand offers students experiences to make mathematical connections, form generalizations, and develop mathematical strategies that lead to making sense of early algebra concepts. Working on projects that deepen the level of mathematical understanding and promote algebra applications has the potential to prepare students for the level of abstraction and symbolism that students need for success in rigorous algebra courses.

Although many individual factors enter into decision about when to offer algebra, explicitly identifying student qualifications that ensure success, teaching for reasoning and sense making at all levels, and striving to give all students a rich and meaningful experience no matter when they take algebra should be high priorities.

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Algebra Resources from Illuminations

NCTM's *Illuminations* website has several algebra-themed resources. Learn how to represent and solve algebra problems using the Algebra Tiles applet. In the Polynomial Puzzler lesson, students use puzzlers to factor polynomials and multiply monomials and binomials. Or perhaps you will find these PanBalance lessons useful!

ATMOPAV IS REACHING OUT TO ELEMENTARY SCHOOL TEACHERS

We recognize that most elementary school teachers (pre-K through 6th or 8th grade) teach in multiple discipline areas. Sustaining several professional memberships can be expensive, but remaining involved with professional development for mathematics is just as important as it is for reading, writing, and social studies. So we are encouraging our local elementary school teachers by offering discounted individual memberships. Math specialists in elementary schools are not eligible for this discount, but general classroom teachers and their schools are urged to take advantage of it.

If you are not an elementary teacher, please pass the word (and the membership application on page 19) on to eligible colleagues who are.

"It has become almost a cliché to remark that nobody boasts of ignorance of literature, but it is socially acceptable to boast ignorance of science and proudly claim incompetence in mathematics."
– Richard Dawkins

TECHNOLOGY CORNER

Trouble With The Curve

A Quick Look at Concavity

Tom Waltrich, Gwynedd Mercy Academy High School
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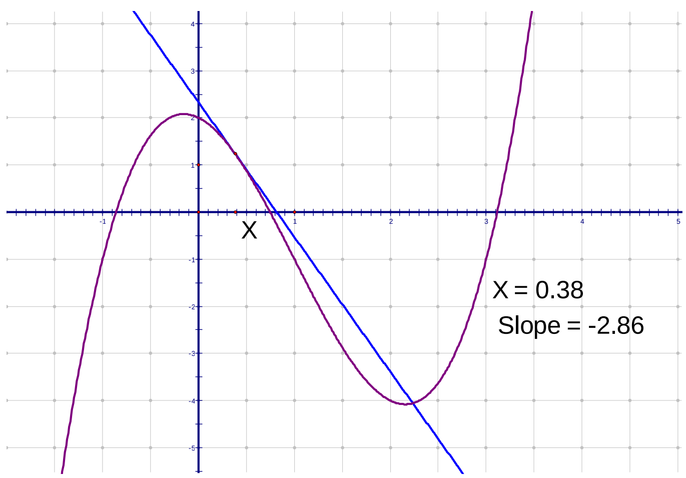
Calculus can be an intimidating subject for many students, but many of the concepts taught can be somewhat intuitive. In my experience, the idea of increasing and decreasing functions is relatively easy for students to comprehend. But the notion of concavity seems less so. Using Sketchpad, students can begin to make the connection between curvature and the second derivative.

I present to the students cubic and quartic functions with the tangent lines drawn in at some point along the x-axis labeled simply as X. The slope is measured and we drag point X to determine the behavior of the slope. The students are asked to describe the behavior of the slope as the tangent line moves along the graph. For some intervals, the slope is increasing and for others the slope is decreasing. I ask my students to take note of where the slopes appear to not change. They make a sign chart and compare it to the original function.



Since the signs do not represent increasing or decreasing, they must indicate something else. Some guidance may be needed to get them to look at curvature, but eventually they will agree that where a graph is curved downward, the slopes are changing negatively and curved upward for slopes changing positively. From here, it is not a difficult leap of faith to see that the change of the slopes is really the change of the change of a function, which is ultimately the 2^{nd} derivative.

Once the students understand that negative values in the second derivative lead to downward curves and positive values lead to upward curves, they can reason through the 2^{nd} derivative test.

Statistics Topics on the TI-84 Plus

Margaret Deckman
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Roman Catholic High School

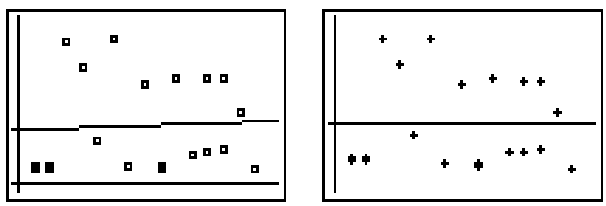
Editor's note: This is a "blast from the past" that is well worth putting out there again.

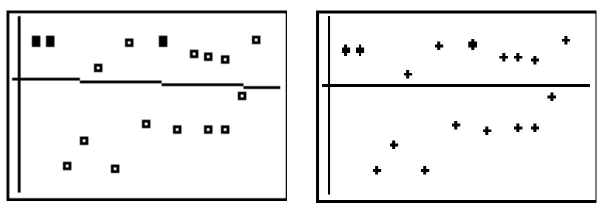
Here are some ideas I found helpful in the understanding of statistics. I don't know what the students thought, but *I* felt better!

I. Scatterplots and Residuals

To take some of the mystery out of residuals, I devised a matching test comparing scatterplots with their residual plots. This was seatwork, but I did create a multiple choice question out of the material for a test.

Anyone who is interested may copy the following. The residuals are paired with their scatterplots here: you can cut, mix, and paste. If you use material from a text, try for a variety of different-looking plots.

1.


2.


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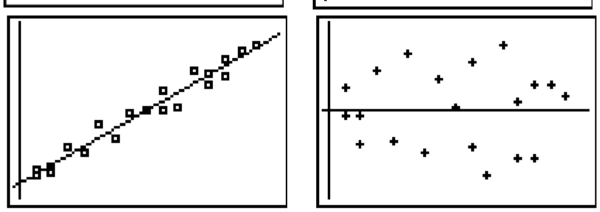

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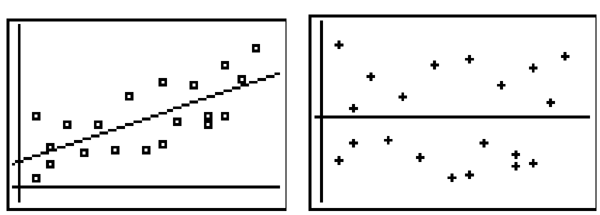

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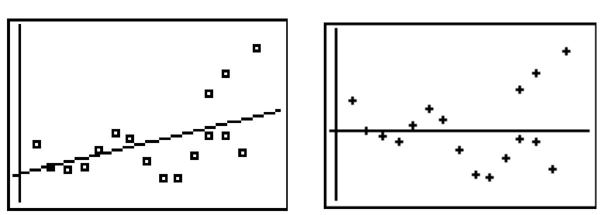
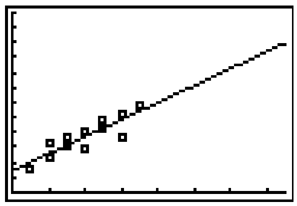


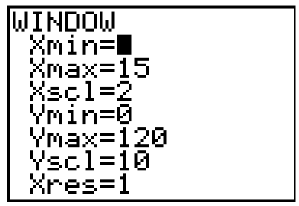
II. Leverage points

To see just how leverage points affect regression equations and scatter, I had the class enter data like this into their calculators:

L1	1	2	2	3	3	4	4	5	5	6	6	7
L2	15	33	24	30	37	29	40	42	45	37	40	58

On the next page is a graph of the scatterplot and regression line, along with the window used (NOT Zoom 9!):

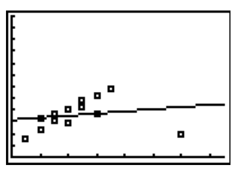
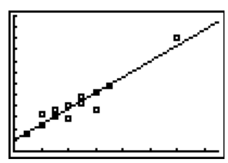
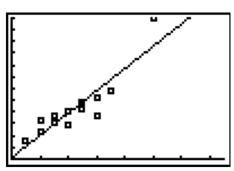
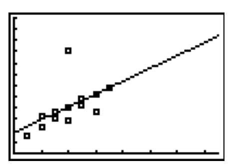




We then added various outliers to see what would happen. The regression equation for each and the correlation, rounded to two decimal places, are given here, and the corresponding graphs after.

Original: $y = 5.69x + 14.35$; $r = 0.86$

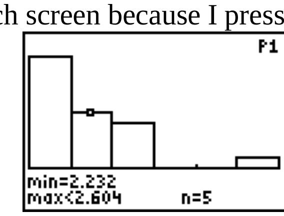
1. (12, 20): $y = 0.93x + 31.50$; $r = 0.21$
2. (12, 100): $y = 7.01x + 9.56$; $r = 0.95$
3. (10,120): $y = 9.48x + 1.17$; $r = 0.90$
4. (4, 90): $y = 5.68x + 18.42$; $r = 0.54$

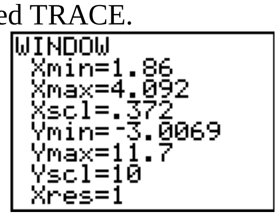
1. 
2. 
3. 
4. 

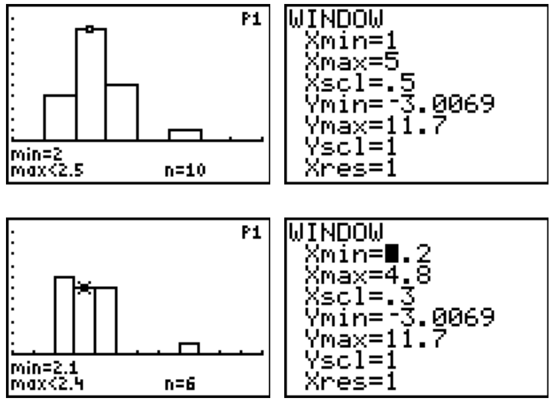
III. Histograms

The histograms generated by the calculator aren't usually very friendly! I like to change the window (after pressing Zoom 9) to get something more decipherable. Actually, the results can look different, as seen here. (There is room for class discussion there.) The first graph was generated by the calculator. I then tinkered with the x-axis for the other histograms. You have to be creative changing the window if you want the boxes to line up with the tick marks. Or let Xscl = 0.

Data are printed at the bottom of each screen because I pressed TRACE.







I hope the rest of the school year is productive and satisfying for everyone!

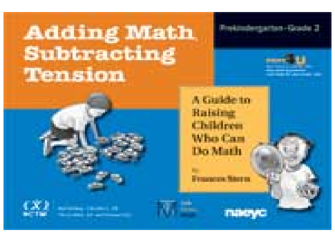
Book Review:

Adding Math, Subtracting Tension

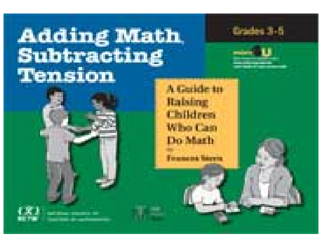
Author: Frances Stern

(2 books: for pre-K to 2nd grade and for 3rd to 5th grade)

These two NCTM publications are intended for parents and are subtitled “A Guide to Raising Children Who Can Do Math.” These would be excellent books to recommend to parents of any young child. Many of the activities and teaching strategies would be useful to classroom teachers as well. They cover a large range of topics and discuss the prerequisite understandings that children need in order to progress. For example, they point out that a child must understand one-to-one correspondence before chanting numbers becomes actual counting. Things are explained in parent-friendly language that offers a clear description of a child’s cognitive maturation and provides mathematical background for the skills and activities that are included.



Both books are extremely well-organized. Sidebar notes add significantly to the main text. They contain cross-references to other parts of the book that may be helpful for the current chapter, explain terms, suggest activities, and more. Each chapter ends with a summary of its important ideas and some additional advice. For example, the chapter on place value’s role in addition and subtraction ends with: *Don’t hurry your child. Rushing to learn the arithmetic algorithms without understanding place value is the beginning of most children’s difficulty with math.* The one criticism I have is that both books lack an index, so a parent who wants to review something such as order of operations is going to have to flip pages to find the sidebar that explains it.



The activities include card games, number tricks and puzzles, directions to make a balance scale from a broomstick and coat hanger, and much more. There are detailed explanations of standard and alternative computation algorithms.

There is a code in the front of each book which allows parents to log onto the NCTM “More4U” site. There, parents can see short annotated videos of children learning and applying math skills and can also download additional printed materials. Sidebar notes sometimes direct parents to that site when it is relevant to the chapter content, such as saying that printable graph paper can be found there.

The books can be ordered from the NCTM website and are also available from online vendors such as Amazon. You might want to have sample copies on hand the next time your school does a Math Night for its parents.

Pedagogy: A Continuing Conversation

Do you have something to share about what, when, and/or how we teach? Contributions are needed for this column.

Numeracy Begins Early . . . or Not?

Lynn Hughes
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In her *President's Message*, Susan Negro referred to a study done by researchers at the University of Missouri which showed the tremendous importance of mathematical experiences that develop core skills and conceptual understanding before first grade. The original study's abstract says:

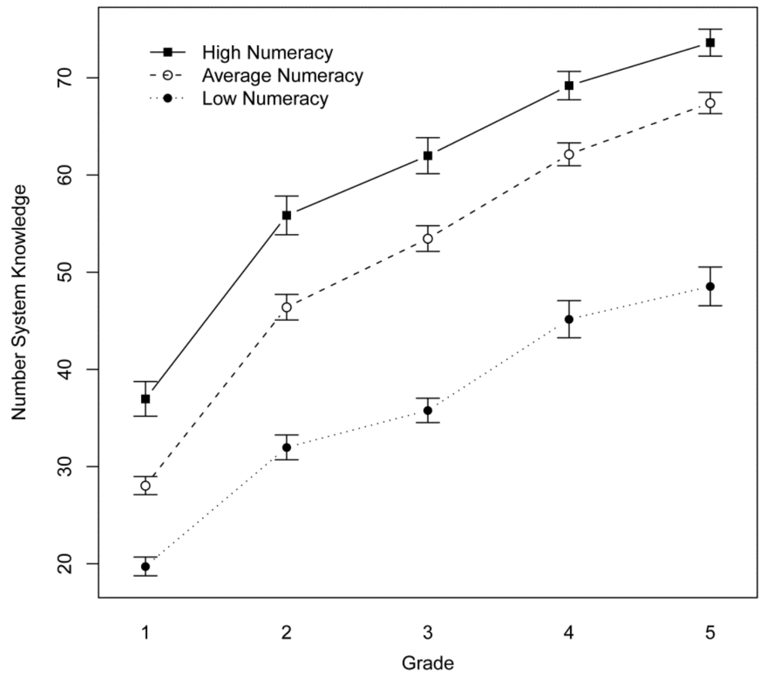
> We administered functional numeracy measures used in studies of young adults' employability and wages to 180 thirteen-year-olds. The adolescents began the study in kindergarten and participated in multiple assessments of intelligence, working memory, mathematical cognition, achievement, and in-class attentive behavior. Their number system knowledge at the beginning of first grade was defined by measures that assessed knowledge of the systematic relations among Arabic numerals and skill at using this knowledge to solve arithmetic problems. Early number system knowledge predicted functional numeracy more than six years later ($ß = 0.195$, $p = .0014$) controlling for intelligence, working memory, in-class attentive behavior, mathematical achievement, demographic and other factors, but skill at using counting procedures to solve arithmetic problems did not. In all, we identified specific beginning of schooling numerical knowledge that contributes to individual differences in adolescents' functional numeracy and demonstrated that performance on mathematical achievement tests underestimates the importance of this early knowledge.

Source: Geary DC, Hoard MK, Nugent L, Bailey DH (2013) Adolescents' Functional Numeracy Is Predicted by Their School Entry Number System Knowledge. PLoS ONE 8(1): e54651. doi:10.1371/journal.pone.0054651. (This and all excerpts from this study that follow are reprinted by permission.)

We know that many children are not enrolled in pre-K and kindergarten programs that develop the learning that the authors of this study believe is essential, and many other children are not in any full-day pre-K or kindergarten program at all. Most of those are likely to be at home or with caregivers that do not or cannot provide appropriate experiences. By the time those students reach secondary school, their degree of success with mathematics would seem to be determined. Can the deficit be made up in first and second grade? Can it be effectively remediated as late as middle school? That is perhaps a subject for a different longitudinal study. But my conjecture is that it is possible, provided that the teachers of those children are aware of the need for foundation experiences that may start at a more basic level than the required curriculum for that grade prescribes. The graph on the following page is from the study named above and describes children's developing numeracy from first through fifth grade. Note that the gap between the "low" group and "average/high" groups increases as the students become older. Is this a function of intelligence, economic class, or ability? The researchers state that they have adjusted for those things in their study and that it is early mastery of foundation skills that is the key. (See the full research report for detailed information.)

What can we, as mathematics educators, learn from this study? One major element is the importance of effective mathematics education in pre-K and kindergarten. How many of those teachers recognize themselves as the gatekeepers of their students' success (and interest) in mathematics 10, 12, or more years in the future? How many are prepared to create the learning environment that their students need? And, of those who are ready and eager to do this, how many are given the administrative support and time in the instructional day to make it happen? How effectively are organizations such as ATMOPAV working to ensure that the curriculum emphasis and teacher training that goes into developing early literacy skills is equaled by the curriculum emphasis on and teacher training for mathematics?

The numeracy study speaks specifically of "relations among Arabic numerals" and "using this knowledge to solve arithmetic problems." However, I think that the requisite foundation knowledge is broader and deeper. A child's sense of number is not dependent on numerals, and children who are not ready to read and write numerals meaningfully are quite capable of developing a sense of number, including such things as one-to-one correspondence, patterns and sequencing, and comparison in terms of greater/lesser/equal. These are the kinds of things that need to happen in children's early years at home and at school if they are going to be successful with and interested in mathematics later on.



There is research (not a part of this study) to suggest that moving children to abstraction before they are ready can prevent the development of secure understanding at a concrete level. Those children build a collection of disconnected rote knowledge that is entirely dependent on memory. They cannot reconstruct what they have forgotten. They have no coherent framework into which new learning can be fitted into an organized context. This is comparable to a young reader who is able to acquire an extensive sight vocabulary but did not master the phonics skills needed to figure out the words that they haven't memorized.

I often think about a student I had many years ago, who was completely lost in her fifth grade math program, although her reading and writing skills were satisfactory. With a lot of extra practice and teacher involvement, she could learn new processes, but she quickly forgot them and was not able to decide whether her strategies and solutions were reasonable. While assessing her understanding of multiplication and arrays, I described to her a room that had 6 rows of chairs. Each row contained 5 chairs. I asked her to calculate how many chairs were in the room. She wrote 6 x 5 = 30. I then asked her to draw the seating arrangement. She drew:

□□□□□□

□□□□□

While this is an extreme example of a student's learning issues in relation to mathematics, it helps to point up the potential disconnect between rote arithmetic skills and true understanding. I wonder what might have changed if I had started her instruction with pre-K math activities instead of trying to fit her into fifth grade work.

The developers of "Singapore Math" (marketed in the USA primarily as the *Math in Focus* program) make a strong case for moving learners from concrete experience to pictorial representation and finally to abstract processes and concepts. This approach is repeated in the K – 8^{th} program for every new topic, not just things at the early-elementary level. This is a very different view from the pedagogical philosophy which holds that a child's maturing brain shifts away from the need for concrete experience and can begin new topics at an abstract level in late elementary or early middle school.

What do NCTM's *Curriculum Focal Points* say about pre-K and kindergarten instruction? In those years, they recommend emphasis on number and operations, geometry, and measurement. Children should develop "an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison" and they learn to "represent, compare, and order whole numbers and joining and separating sets." They learn to "identify shapes and describe spatial relationships" in two and three dimensions." They learn to identify "measurable attributes and compare objects using these attributes." Activities that develop these understandings should incorporate problem-solving, reasoning, and communication. We can see how all of this constitutes readiness for later instructional experiences and is likely to lead to serious difficulty if it is not securely understood.

The irony is that most young children integrate much of this into their self-selected play. They sort, stack, compare, and count objects; build patterned trains of colorful blocks, pour water into containers of various sizes, and draw their world on paper. What it needs is for adults to provide time and materials to create a lot of opportunity for that discovery and exploration, recognize the importance of it (including the child's desire to do the same things again and again), and watch for children who may need some extra support in making explicit sense of what they are doing – including the development of mathematical vocabulary. Teachers need to do this, and they should have conversations with parents to ensure that it is also happening at home. Many parents read to their children; not as many intentionally do things that develop math skills. (See the book review in this issue for two publications that would be tremendously helpful to parents.)

One of the most effective programs for assisting students who have struggled with learning to read is the Wilson Reading Program, founded on principles developed by Orton-Gillingham. Whether the student in the program is a fourth grader or an adult, the instructional plan is the same. It starts with the most basic elements of reading and moves forward in sequence, skipping nothing. This may be something to consider when we work with a student who is floundering in mathematics. The difficulty may arise from things the student did not master (or encounter) in those earliest years. There is not much point in trying to teach scientific notation if a student doesn't understand place value, our base-ten system, and orders of magnitude. Instruction may need to begin at the beginning again, not simply be intensified in the middle.

That brings us back to those elementary school classrooms. If a child is running into difficulty in second or third grade, it may be effective to go back to those pre-K and kindergarten activities instead of sending home more practice worksheets. Drilling on a subtraction algorithm may yield narrow success, but we still have students in fifth and sixth grade who don't remember to regroup where needed and ignore or reverse the troublesome digits instead. Further, many students at that level don't seem to know that the number being subtracted is already contained in the number they have and think of the numbers as two discrete quantities (as they would be for addition). Continuous use of concrete materials as well as pictorial models and diagrams (such as bar modeling) could serve to reveal and help to remediate misunderstandings such as these in upper elementary and middle school.

Below are some conclusions from the study cited at the start of this article:

- The results provide three key insights into children's mathematical development. The first is that some aspects of their school entry quantitative knowledge, as measured by the mathematical cognition tasks, contribute to long-term functional numeracy, controlling other factors that affect learning, whereas other aspects of their knowledge do not.
- At the same time, children's skill at using counting procedures to solve addition problems at the beginning of first grade was not predictive of their later functional numeracy scores, holding other factors constant.
- The second key finding is the previously noted relation between mathematics achievement in kindergarten and mathematics achievement throughout schooling may underestimate the long-term consequences of poor school entry quantitative knowledge.
- The third key finding is that growth in number system knowledge is less important for predicting functional numeracy than is school entry number system knowledge.
- For now, the implication is that interventions to improve children's early understanding of the relations among numerals need to be implemented before the start of schooling or in first grade, and fortunately such interventions are being developed.

You are strongly encouraged to read the full report, found here:

http://www.plosone.org/article/info:doi/10.1371/journal.pone.0054651

If you are an elementary teacher, please encourage your colleagues to join ATMOPAV, PCTM, and NCTM, especially those teaching at the youngest levels. Our membership fees are very reasonable. Consider offering a workshop at an upcoming conference, and persuade colleagues to do the same. This study illustrates the far-reaching importance of meaningful mathematical experiences that are needed long before those students start to grapple with Algebra I.

If you have already renewed, how about passing this on to a colleague that is not yet a member?

ATMOPAV MEMBERSHIP APPLICATION

Membership runs from September through June of the current academic year. To renew your membership or register as a new member, include a check payable to **ATMOPAV** along with this completed form and mail to:

Allison Rader
ATMOPAV Membership
1207 Stonybrook Drive
Norristown, Pa. 19403

Check your membership dues category and complete the information, to update our membership records, below:

☐ Regular annual membership **$10**

☐ Elementary (pre-K to 6) annual membership (Special Promotion) **$5**

☐ 5-year membership **$40**

☐ Life membership **$100**

☐ Student membership (Full-Time Undergraduate) **free**

Circle all that apply.

Teacher Chairperson Supervisor Administrator Retired Other ________________

Elementary Middle School/JR High SR High College Other _________________

Please **print clearly,** filling in ***all*** of the blanks.

Name:____

e-mail:____ Home Telephone:________________________

Mailing Address:____
(# and Street Address)

____
(City) (State) (Zip code)

School:___________________________________ School District:_________________________________

Please check if you have membership in: ☐ PCTM ☐ NCTM

Information about ATMOPAV and Spring/Fall Conferences can be found at www.atmopav.com